

Position Accuracy Analysis of the Nominal DCATT Hexapod Design

by
BRENDON PERKINS

03/01/99

1.0 INTRODUCTION

The DCATT hexapod consists of six actuators arranged in an elephant stand type configuration known as a Stewart platform. Stewart Platforms are traditionally employed in robotic and machine tool applications that require motion in all six degrees of freedom. In fact, there is extensive literature regarding the development of Stewart platforms for these types of applications in which the most predominant design considerations are absolute position accuracy, range of motion, as well as the identification and elimination of singularity points. Unlike the more traditional applications, the DCATT hexapod design is governed by a somewhat different set of performance considerations.

Since the DCATT hexapods will be used for producing optical alignments, the required range of motion can be quite small. This, in turn, eliminates the occurrence of singularity points. While absolute position accuracy is an important consideration for collision avoidance between the segments of the primary mirror, none of the DCATT operational phases requires absolute position knowledge of the hexapods to produce convergence to the best-aligned state. Nevertheless, the control code of the fine alignment phase produces relative positions for each hexapod based on information it gathers from a pupil image. Therefore, the relative position accuracy of the nominal design for the DCATT hexapods is expected to be the most critical design constraint. Repeatability of relative displacements is also a concern.

Within the context of this document, the term "position" will be used to refer to both angular and linear positions. As such, it can be broken into the components of relative piston, pointing, and decenter accuracy. Of these three components, the relative pointing accuracy is the most significant. In fact, it's expected that the DCATT system will require each hexapod to consistently produce a relative pointing accuracy of approximately $3.1\text{E-}3$ arc-s.

2.0 PURPOSE

This document presents mathematical descriptions for both absolute and relative position accuracy. It also presents the conclusions and results of an analysis that was performed to determine the position accuracy of the nominal design for the DCATT hexapods based on a consideration of actuator length errors and resolution limits. It's important to note that this document does not present the actual position accuracy that is to be expected from each hexapod. An estimate of the actual position accuracy can only be determined after all error sources have been accounted for. Additional analyses will soon be performed to account for the effects of gravity, internal hexapod stresses, optical misalignments, and measurement errors during hexapod characterization. Comprehensive characterization tests of the individual actuators may also be required to verify their actual resolution limits as a function of temperature, load, hysteresis, displacement direction, and velocity.

3.0 ANALYSIS CONCLUSIONS

Comprehensive maps and tabulated values for the absolute position accuracy of the nominal hexapod design are presented in Section 10.0. This data shows the average, absolute pointing accuracy to be 0.418 arc-s; the average, absolute decenter accuracy to be 2.5E-7 mm; and the absolute piston accuracy to be 1.4E-4 mm. The actual pointing accuracy has a variation of less than 1.0% from its average value over the entire range of motion. Similarly, the actual decenter and piston accuracies exhibit variations of less than 2.0% and 0.5% respectively.

Comprehensive maps and tabulated values for the relative position accuracy are presented in Section 11.0. The relative pointing accuracy was found to be greater than the resolving power of the analysis software (using double precision calculations) over the entire range of motion. The resolving power of the analysis software is 5.1E-3 arc-s. This analysis also shows the average, relative decenter and piston accuracies to be 8.3E-6 mm and 6.6E-6 mm. Although nothing more can be said of the relative pointing accuracy, the relative decenter and piston accuracies exhibit variations of less than 1.3% and 0.5%, respectively, over the hexapod's entire range of motion.

4.0 NOMINAL HEXAPOD DESIGN

The analysis presented herein relies extensively on the use of a rigid body model of the nominal hexapod design. When all the actuators of the nominal design are moved to their midrange positions, the hexapod is said to be in its home state. When placed in the home state, the nominal hexapod design has five defining characteristics. They are as follows:

1. All actuators reside at their midrange extensions with total lengths of 208.9 mm,
2. All actuator base connections lie in a single plane (the base plane) and form a circle with a radius of 130.85 mm,
3. All actuator platform connections lie in a plane (the platform plane) and form a circle with a radius of 92.04 mm,
4. The base plane and platform planes are parallel, and
5. A line that passes through the centers of both circles is also perpendicular to both planes.

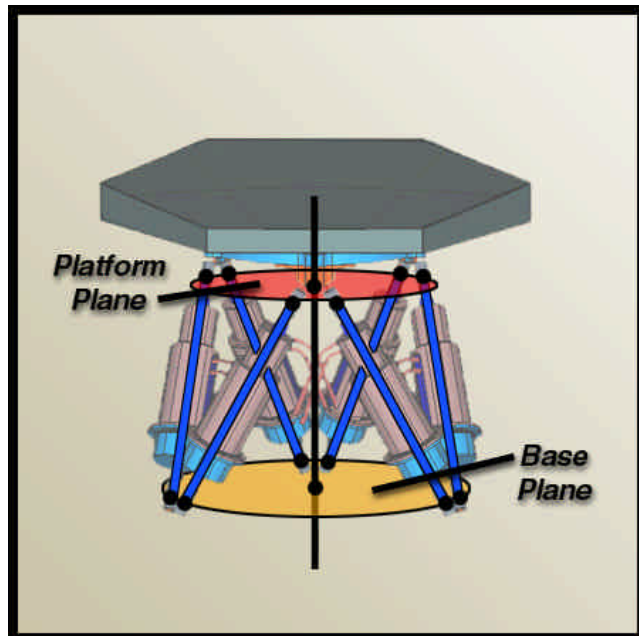


Figure 1: Nominal Hexapod Design

Although it's preferable for the integrated hexapods to produce the aligned state of the system when they're all in their home positions, this is not required for the system to converge to its best-aligned state during normal operations. It should also be noted that the actual geometry of each hexapod will vary from the nominal design because of internal misalignments and manufacturing tolerances. In fact, the intent of the hexapod characterization process is to determine the actual geometry of each hexapod. This knowledge will then be used to develop models for use with the rigid body kinematics to accurately control the hexapods.

5.0 HEXAPOD COORDINATE FRAMES AND NUMBERING SCHEMES

To properly interpret the significance of the results and conclusions presented herein, it's important to know the two coordinate systems that the kinematics routines use to complete their calculations. These two coordinate systems, referred to as the base and platform frames, are shown in Figure 2 along with the actuator numbering scheme. The origin of the base frame is centered 22.36 mm below the base plane. Its z-axis is defined to be perpendicular to the base plane and passes through the center of the base circle. The +x-axis is defined to be parallel to the base plane and points radially outward from the vertex of the parent mirror when the hexapod is integrated with the rest of the system. The platform frame has the same orientation but is centered 22.36 mm above the platform plane at the vertex of the attached mirror segment.

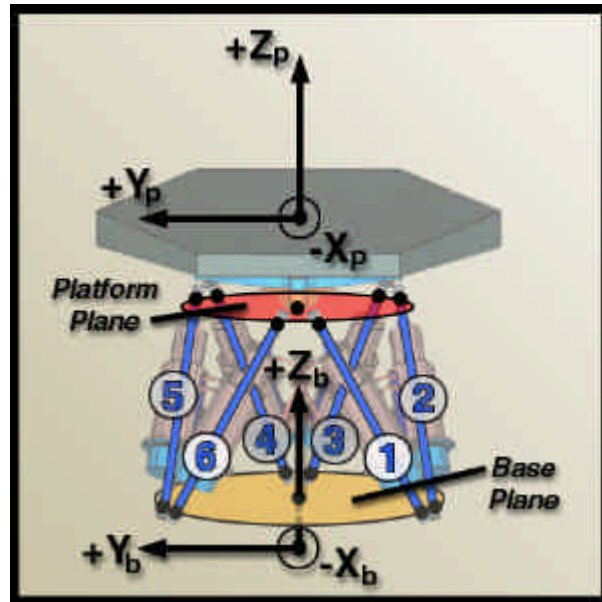


Figure 2: Hexapod Framework & Numbering

The base frame essentially serves as the hexapod's global coordinate system while the platform frame provides a convenient framework for specifying segment positions in terms of optical coordinates about the vertex. When used with the optical model, it should be noted that the platform frame is specified as the element coordinate system in the MACOS prescription file, and is often defined with a slightly different orientation such that movements along the z-axis produce pure piston

6.0 HEXAPOD KINEMATICS ROUTINES

Fortunately the kinematics routines are extremely simple and easy to understand since they involve nothing more than a few matrix operations. In fact, the kinematics code consists of only two routines. The first routine is referred to as the inverse kinematics solution, and it's used to convert segment dof's into actuator lengths. The second routine is referred to as the forward kinematics solution, and it's used to convert actuator lengths into segment dof's. Within the context of this document, a conceptual description of these routines will suffice.

Conceptual Description of the Inverse Kinematics Solution

Whenever a new segment position is to be produced, the inverse kinematics routine implements a three step process in which it: 1) transforms the platform points to the new position, 2) translates the platform points (which are defined in the platform frame) into their equivalent base coordinates, and 3) subtracts the base connection points from the platform connection points to reveal the new actuator lengths that are desired. In the real hardware, the actuators are then commanded to move to new positions that produce the desired lengths and, in turn, the desired position.

Regardless of whether a hexapod position is specified in terms of relative or absolute coordinates, the rigid body approach always relies on knowledge of absolute geometry to produce a new position of the hexapod. More specifically, the absolute positions of all the base and platform connection points must be known to calculate the lengths of the actuators.

Conceptual Description of the Forward Kinematics Solution

Unlike the inverse kinematics solution, the forward kinematics solution implements an iterative approach in which it feeds multiple, intelligent guesses for the segment dof's into the inverse routine until the inverse routine produces values that match the known actuator lengths to within some desired tolerance. The final guess then represents the sought after dof's.

7.0 ASSUMPTIONS

For the purpose of this analysis, it's assumed that absolute position knowledge during normal operation of the hexapod will be limited by the accuracy of the limit switch located at each actuator's end-of-range position corresponding to full contraction. This error is introduced after hexapod characterization when the actuators are turned off. It cannot be assumed that a hexapod will retain its position after loss of power. Therefore, the actuators of each hexapod must be moved to their fully contracted positions to regain absolute position knowledge as soon as they are powered on. Absolute position knowledge will then be limited to the accuracy of the limit switches, which is currently stated to be +/-125 nm. Nevertheless, each actuator will be able to produce relative movements on the order of 2.9 nm. It should also be noted that the limit switch error represents a systematic error since it does not change until the actuators are moved back on and off their limit switches, and this will seldom be done.

8.0 HEXAPOD POSITION ACCURACY

To better understand the significance of the conclusions and analytical results presented herein, it's important to understand exactly what is meant by absolute and relative position accuracy. These concepts are presented in mathematical form.

Consider Figure 3, which shows the nominal design of the hexapod in a position other than its home state. For the sake of convenience, a point, \mathbf{s} , and a unit vector, \mathbf{P} , will be used as indicators of the hexapod's linear position and pointing direction. By definition, the point is located at the vertex of the mirror segment and the pointing vector is perpendicular to the platform plane when the hexapod is placed in its home position. As the segment is moved from one position to the next, the point and the pointing vector move with the segment.

Absolute Position Accuracy

Consider Figure 3, which shows an exaggerated, close-up view of the attached mirror segment placed in a position other than its home state. It's important to note that the point, \mathbf{s} , and the pointing vector, \mathbf{P} , are actually idealized since hexapod misalignments, machining tolerances, measurement errors, and other systematic errors will cause the hexapod to assume a position that is different from the assumed (ideal) position. In fact, the difference between the actual and assumed positions is the absolute position error. More specifically, the absolute linear position error, $d\mathbf{s}_{err}$, is simply the difference between the actual linear position, \mathbf{s}_{actual} , and the assumed position, \mathbf{s}_{ideal} . Similarly, the pointing error, $d\mathbf{P}_{err}$, is the difference between the actual and assumed pointing vectors, $\mathbf{P}_{actual} - \mathbf{P}_{ideal}$ (for small angles) and the dot product of these vectors for large angles.

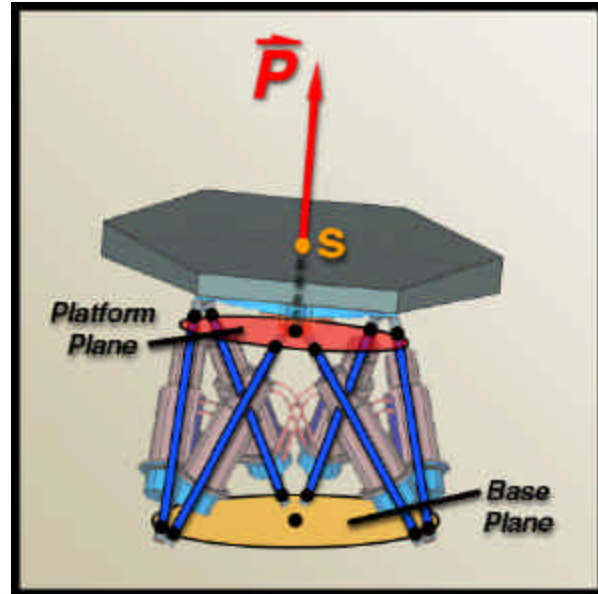


Figure 3: Hexapod Pointing Vector

In Figure 4 the sphere represents the boundary of maximum, linear position error which can be broken down into its components of decenter and piston. The circle represents the boundary of maximum decenter error while the +/- z-axis extremes of the sphere represent the boundaries of maximum piston error. The coordinate frame that is shown represents the platform coordinate frame.

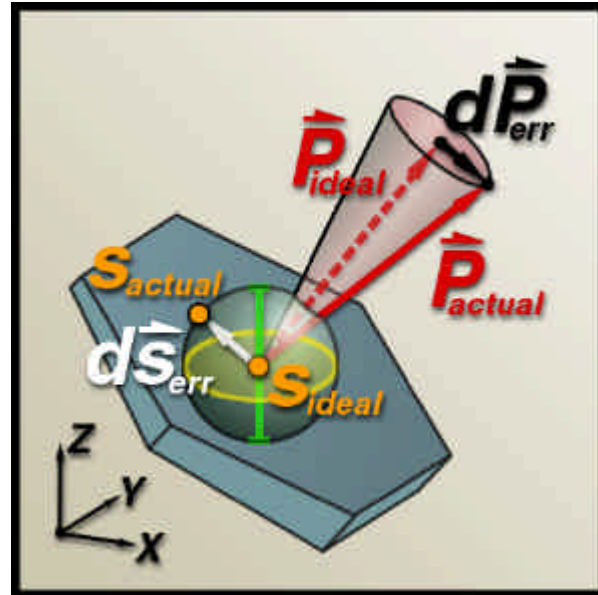


Figure 4: Absolute Pointing, Decenter & Piston Error

As previously mentioned, the actuator limit switch error is by far the largest source of uncertainty in determining the actuator lengths. By considering all the +/- combinations of this error source (+/- 125 nm) over all six actuators in the hexapod, it's possible to determine the hexapod's minimum position error as a function of actuator length uncertainties. In fact, the analysis presented herein was implemented to evaluate position uncertainty at incremental positions throughout the hexapod's entire range of motion. The implementation details of this analysis are described in Section 9.0 while the resulting error maps and tabulated values are presented in Section 10.0.

Relative Position Accuracy

Fortunately, absolute position errors are not a problem since none of the alignment operations rely on absolute position knowledge to produce convergence to the aligned state of the system. In fact, the fine alignment control algorithm will provide relative position commands to the controller of the optical telescope assembly. Nevertheless, the internal, rigid-body kinematics routines do rely on knowledge of absolute geometry as previously discussed in Section 6.0. Therefore, any uncertainty in knowledge of a hexapod's absolute geometry will result in errors even when implementing movements to relative positions, and these errors must be accounted for. For the sake of convenience in the ensuing text, the relative linear-position error will be described separately from the relative pointing error.

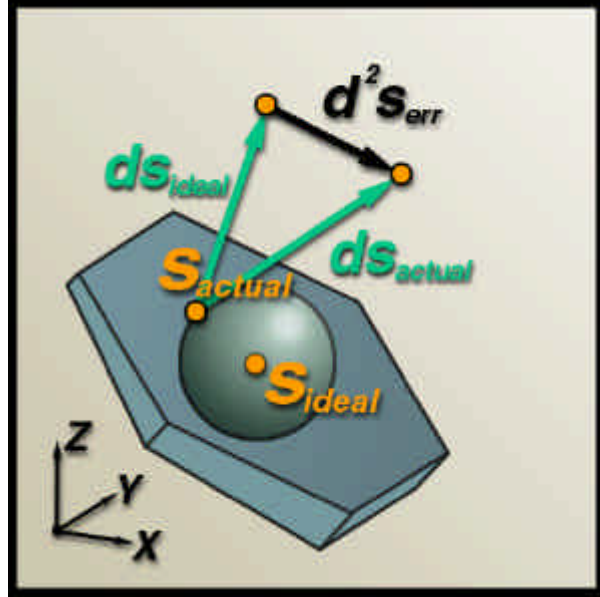


Figure 5: Relative Piston & Decenter Error

Consider Figure 5, which shows the relative linear position error more explicitly in vector form. When implementing relative displacements, the kinematics routines in the hexapod control code will calculate new actuator lengths to produce the desired displacement based on the assumption that the hexapod is in a particular position, s_{ideal} . However, if the hexapod is actually in a different position, s_{actual} , the calculated actuator lengths will produce an actual displacement, ds_{actual} , that is distinctly different from the intended displacement, ds_{ideal} . Therefore, the relative pointing error is the dot product of these two displacement vectors which can be approximated by the difference, d^2s_{err} , between the two, normalized displacement vectors.

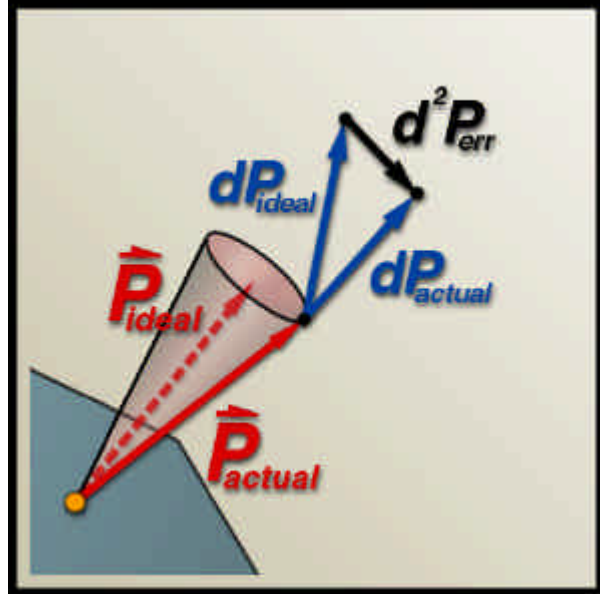


Figure 6: Relative Pointing Error

Now Consider Figure 6. Similarly, a relative pointing error, d^2P_{err} , will arise as the difference between the assumed, relative pointing direction, dP_{ideal} , and the actual, relative pointing direction, dP_{actual} . In other words, the hexapod kinematics will calculate new actuator lengths to produce a displacement pointing direction, dP_{ideal} , based on the assumption that the hexapod is already pointing in a particular direction, P_{ideal} . Since the hexapod will actually be pointing in a different direction, P_{actual} , the calculated actuator lengths will produce a distinctly different pointing displacement, dP_{actual} .

Before proceeding, it should be noted that the relative and absolute position errors are directly proportional. It's also important to note that the relative error is directly proportional to the magnitude of the desired displacement. In other words, small displacements of the hexapod will, in turn, produce small relative position errors. As a result, successive iterations in the fine alignment phase of the DCATT system are expected to result in ever-decreasing, relative displacements of the hexapods. Therefore, the practical limit of relative pointing error will effectively be determined by the average magnitude of commanded displacements when successive iterations do not lead to further improvements in the aligned state of the system. Since this average displacement magnitude is not currently known, the analysis presented herein used the currently stated actuator resolution limit of 2.9 nm to determine the theoretical limit of the hexapod's relative pointing accuracy. The implementation of this analysis is described in Section 9.0 and the results are presented in section 11.0.

9.0 ANALYSIS IMPLEMENTATION

Incremental Hexapod Positions and Range Limits

In order to complete the analysis, the hexapod was exercised throughout its entire range using a 3 coordinate position specification consisting of a piston term (T_{piston}), a radial pointing angle term (θ_{radial}), and an azimuth pointing angle term (θ_{azimuth}). The hexapod was moved incrementally to individual positions using three nested loops (one for each coordinate). The ranges and increment sizes that were used for each coordinate were as follows.

T_{piston} = Range: -3 - +3 mm
Increment: 0.1 mm

θ_{radia} = Range: 0 - 3.50 deg
Increment: 0.05 deg

θ_{azimuth} = Range: 0 - 360 deg
Increment: 15 deg

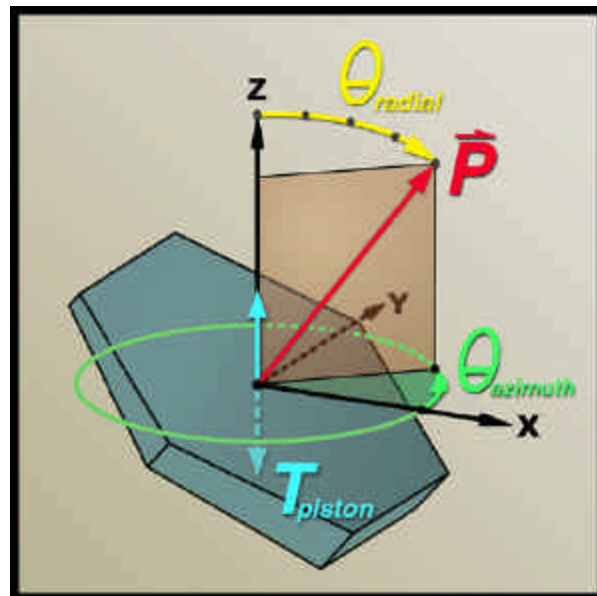


Figure 7: Hexapod Position Coordinates

Determination of Absolute Position Error

At each incremental position, the absolute position error was evaluated by first calculating the equivalent actuator lengths for the position and then adjusting those lengths by all +/- combinations (64 in total) of the limit switch error (+/- 125 nm). Each combination of altered actuator lengths was then fed back through the forward kinematics routine, and the resulting set of altered positions was evaluated against the non-altered position to determine the maximum pointing, piston, and decenter errors. Finally, this data was plotted in a series of maps for the entire operational range of the hexapod, and these maps are provided in Section 10.0.

Determination of Relative Position Error

At each incremental position, the relative pointing error was evaluated by first calculating the equivalent actuator lengths for the position and then adjusting those lengths by all +/- combinations of a single motor step (2.9 nm). The set of the 64 combinations was then used to represent all the possible, ideal, relative movements that could result from actuator length changes of a single step. The analysis then proceeded in the same manner to adjust the actuator lengths for each of the 64 absolute error positions to produce 64 sets of 64 values (4096 total cases) representing all the possible, actual, relative movements that could result from actuator length changes of a single step. The ideal displacement set was then subtracted (on a term by term basis) from the actual displacement sets to produce 4096 relative position errors. The maximum values for each component of the relative position error were then extracted from this set and reported as the component position errors for that incremental position.

10.0 ANALYTICAL RESULTS – ABSOLUTE POSITION ACCURACY

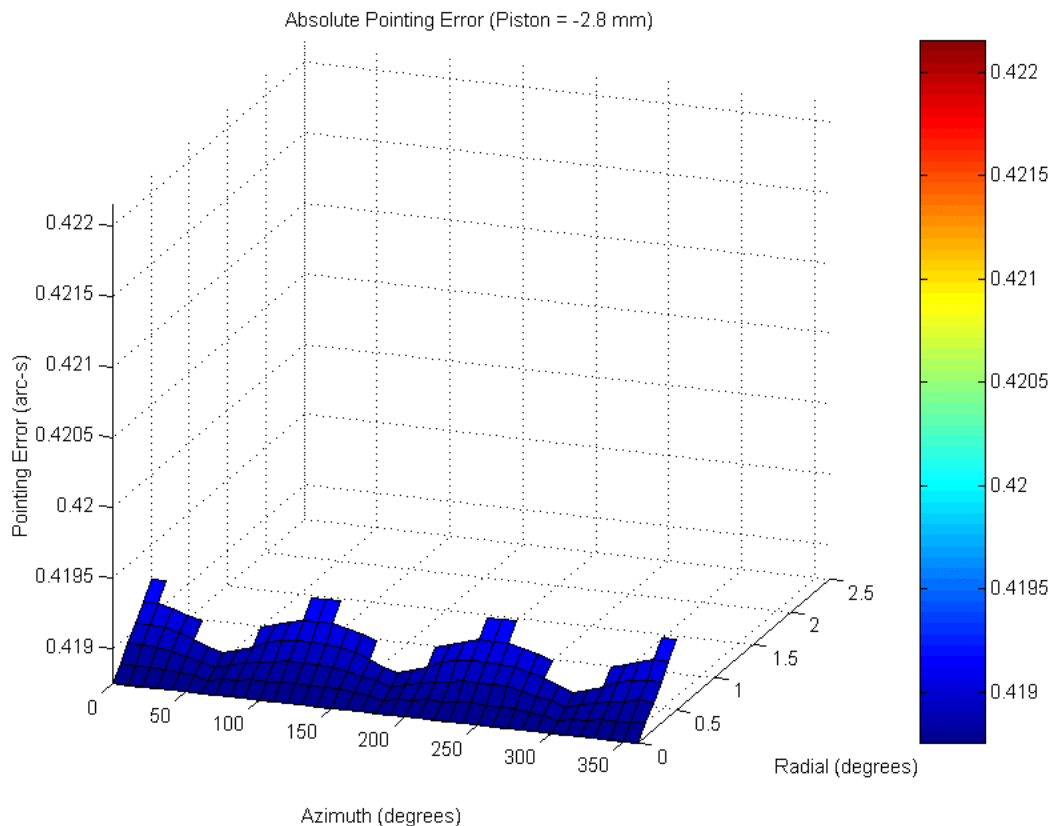
The following table shows the minimum, maximum, and average absolute position error in terms of its constituent components of pointing, decenter, and piston. Each row represents an incremental position of the hexapod in piston. At each increment of piston, the hexapod was then moved through incremental values of radial and azimuth pointing over the full range of motion to evaluate the hexapod position error. The results are tabulated below.

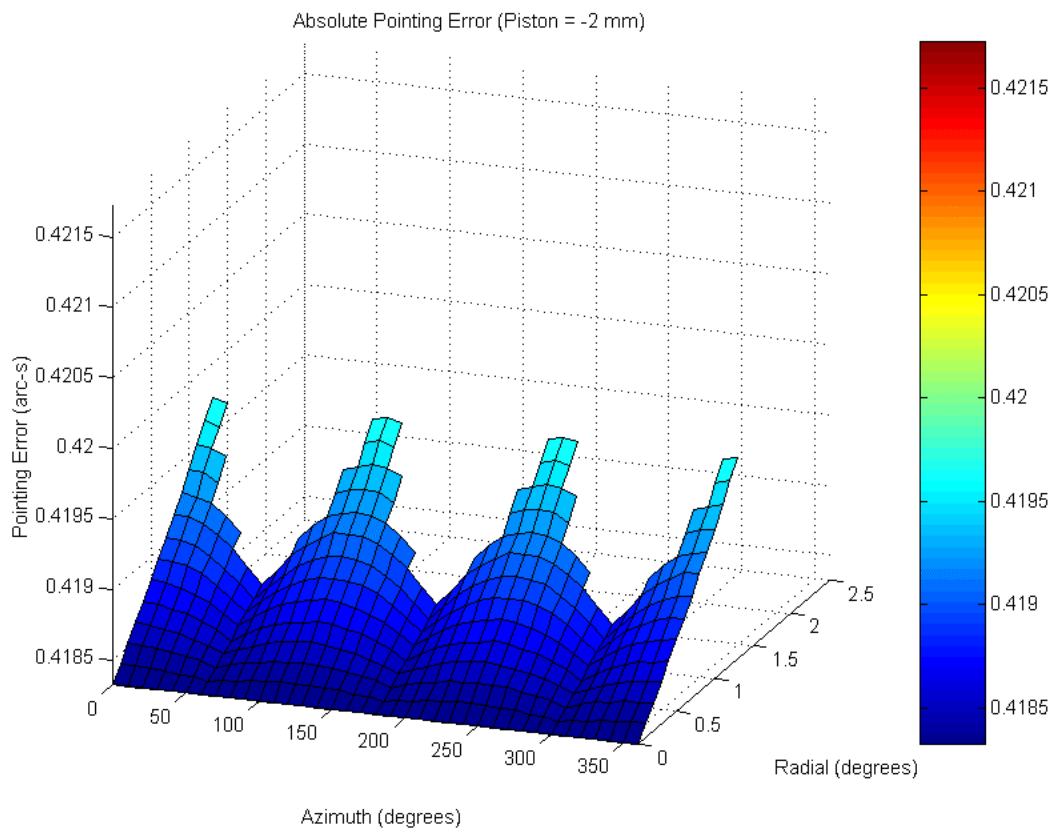
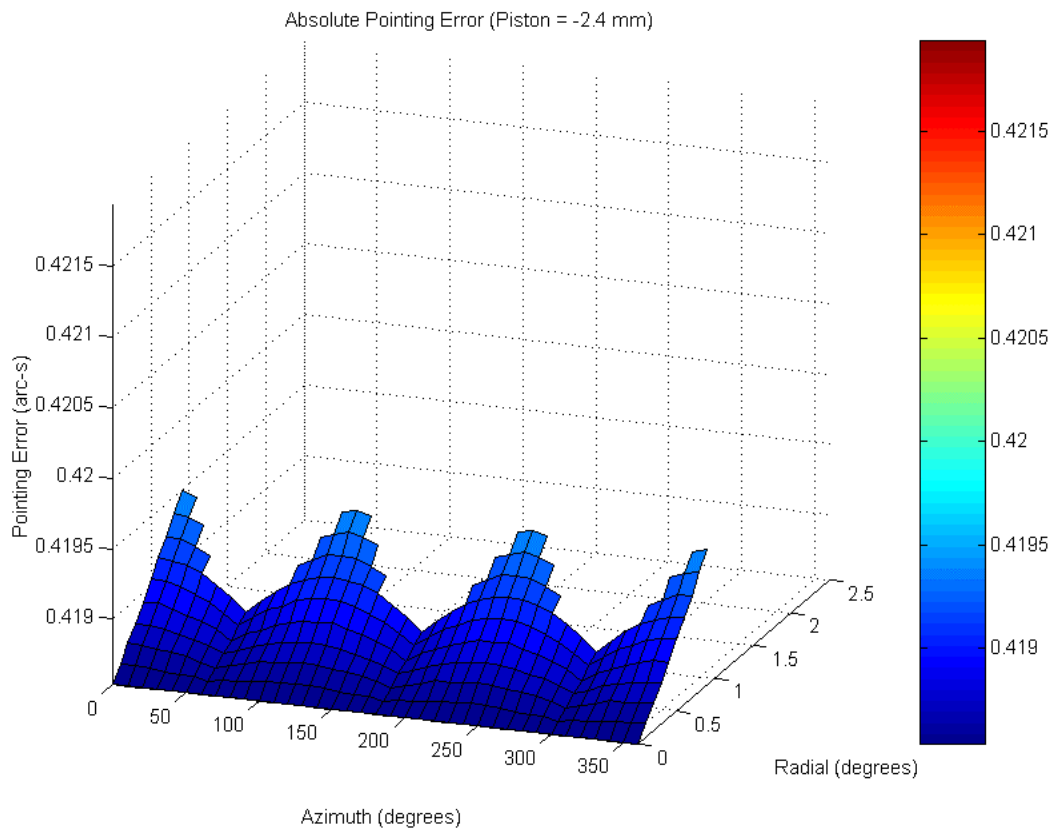
PISTON (mm)	POINTING ERROR			DECENTER ERROR			PISTON ERROR		
	MINIMUM (arc-s)	MAXIMUM (arc-s)	AVERAGE (arc-s)	MINIMUM (mm)	MAXIMUM (mm)	AVERAGE (mm)	MINIMUM (mm)	MAXIMUM (mm)	AVERAGE (mm)
-3.4	4.19068-1	4.19068-1	4.19068-1	3.47888-4	3.47888-4	3.47888-4	1.42626-4	1.42626-4	1.42626-4
-3.3	4.19012-1	4.19114-1	4.19033-1	3.47115-4	3.09017-8	6.57345-9	1.42607-4	1.42607-4	1.42607-4
-3.2	4.18956-1	4.19170-1	4.19021-1	8.73953-7	6.19261-8	2.09359-8	1.42589-4	1.42589-4	1.42589-4
-3.1	4.18911-1	4.19215-1	4.18998-1	3.48275-4	9.30730-8	3.05444-8	1.42571-4	1.42571-4	1.42571-4
-3.0	4.18854-1	4.19373-1	4.18984-1	3.48404-4	1.55839-7	4.45960-8	1.42553-4	1.42553-4	1.42553-4
-2.9	4.18798-1	4.19429-1	4.18972-1	3.42784-0	1.87373-7	5.91863-8	1.42534-4	1.42535-4	1.42534-4
-2.8	4.18753-1	4.19474-1	4.18951-1	2.91432-0	2.19027-7	6.93391-8	1.42516-4	1.42517-4	1.42516-4
-2.7	4.18696-1	4.19542-1	4.18934-1	3.48791-4	2.50803-7	8.22594-8	1.42498-4	1.42499-4	1.42498-4
-2.6	4.18640-1	4.19598-1	4.18918-1	3.48920-4	2.82699-7	9.43582-8	1.42480-4	1.42481-4	1.42480-4
-2.5	4.18595-1	4.19778-1	4.18915-1	1.97549-0	3.47089-7	1.09429-7	1.42462-4	1.42464-4	1.42462-4
-2.4	4.18538-1	4.19845-1	4.18903-1	3.49179-4	3.79367-7	1.23954-7	1.42444-4	1.42446-4	1.42444-4
-2.3	4.18482-1	4.19913-1	4.18877-1	3.49308-4	4.11765-7	1.32282-7	1.42426-4	1.42429-4	1.42426-4
-2.2	4.18425-1	4.19992-1	4.18864-1	1.71789-0	4.44281-7	1.45266-7	1.42408-4	1.42411-4	1.42408-4
-2.1	4.18380-1	4.20183-1	4.18868-1	3.49566-4	5.10018-7	1.63297-7	1.42390-4	1.42394-4	1.42391-4
-2.0	4.18324-1	4.20250-1	4.18852-1	3.49695-4	5.42912-7	1.74172-7	1.42372-4	1.42377-4	1.42373-4
-1.9	4.18267-1	4.20329-1	4.18840-1	3.04840-0	5.75924-7	1.87187-7	1.42354-4	1.42360-4	1.42355-4
-1.8	4.18222-1	4.20408-1	4.18824-1	3.49954-4	6.09053-7	1.98636-7	1.42336-4	1.42342-4	1.42337-4
-1.7	4.18166-1	4.20486-1	4.18814-1	3.50083-4	6.42299-7	2.11876-7	1.42318-4	1.42325-4	1.42319-4
-1.6	4.18109-1	4.20700-1	4.18812-1	3.50213-4	7.09629-7	2.26652-7	1.42300-4	1.42309-4	1.42302-4
-1.5	4.18064-1	4.20778-1	4.18805-1	3.50342-4	7.43248-7	2.39324-7	1.42282-4	1.42292-4	1.42284-4
-1.4	4.18008-1	4.20868-1	4.18798-1	1.83568-0	7.76983-7	2.53758-7	1.42265-4	1.42275-4	1.42266-4
-1.3	4.17962-1	4.20969-1	4.18776-1	2.65360-0	8.10834-7	2.62021-7	1.42247-4	1.42258-4	1.42249-4
-1.2	4.17906-1	4.20913-1	4.18754-1	3.24350-0	8.10278-7	2.71760-7	1.42229-4	1.42240-4	1.42231-4
-1.1	4.17849-1	4.20857-1	4.18735-1	2.81109-0	8.09722-7	2.83051-7	1.42211-4	1.42223-4	1.42214-4
-1.0	4.17804-1	4.20801-1	4.18700-1	3.50988-4	8.09168-7	2.88792-7	1.42194-4	1.42205-4	1.42196-4
-0.9	4.17748-1	4.20745-1	4.18675-1	2.71054-0	8.08615-7	2.97182-7	1.42176-4	1.42187-4	1.42178-4
-0.8	4.17691-1	4.20688-1	4.18650-1	3.51248-4	8.08062-7	3.06365-7	1.42159-4	1.42169-4	1.42161-4
-0.7	4.17646-1	4.20643-1	4.18630-1	3.51377-4	8.07510-7	3.14800-7	1.42141-4	1.42152-4	1.42144-4
-0.6	4.17601-1	4.20453-1	4.18570-1	2.39116-0	7.72742-7	3.14918-7	1.42123-4	1.42133-4	1.42126-4
-0.5	4.17544-1	4.20351-1	4.18530-1	3.51636-4	7.66357-7	3.19191-7	1.42106-4	1.42116-4	1.42108-4
-0.4	4.17487-1	4.20183-1	4.18488-1	1.39959-1	7.49612-7	3.23710-7	1.42088-4	1.42098-4	1.42091-4
-0.3	4.17442-1	4.20014-1	4.18412-1	3.51895-4	7.16882-7	3.19736-7	1.42071-4	1.42080-4	1.42073-4
-0.2	4.17386-1	4.19913-1	4.18363-1	3.75565-0	7.16399-7	3.22059-7	1.42053-4	1.42063-4	1.42056-4
-0.1	4.17340-1	4.19789-1	4.18307-1	3.52155-4	6.95063-7	3.22010-7	1.42036-4	1.42045-4	1.42039-4
+0.0	4.17284-1	4.19688-1	4.18258-1	3.42544-0	6.94599-7	3.24486-7	1.42018-4	1.42028-4	1.42021-4
+0.1	4.17238-1	4.19542-1	4.18187-1	1.96337-0	6.94136-7	3.20174-7	1.42001-4	1.42010-4	1.42004-4
+0.2	4.17182-1	4.19361-1	4.18112-1	3.52544-4	6.72231-7	3.16939-7	1.41984-4	1.41993-4	1.41986-4
+0.3	4.17137-1	4.19283-1	4.18059-1	3.52674-4	7.00137-7	3.18101-7	1.41966-4	1.41977-4	1.41969-4
+0.4	4.17080-1	4.19159-1	4.17982-1	3.20323-0	6.71346-7	3.11946-7	1.41949-4	1.41958-4	1.41952-4
+0.5	4.17023-1	4.18990-1	4.17899-1	3.72236-0	6.81656-7	3.05432-7	1.41932-4	1.41942-4	1.41934-4
+0.6	4.16978-1	4.18888-1	4.17840-1	6.98868-0	6.81212-7	3.02943-7	1.41915-4	1.41925-4	1.41917-4
+0.7	4.16921-1	4.18741-1	4.17759-1	2.31812-0	7.07452-7	2.97717-7	1.41897-4	1.41908-4	1.41900-4
+0.8	4.16876-1	4.18640-1	4.17680-1	3.53323-4	6.99894-7	2.88684-7	1.41880-4	1.41891-4	1.41882-4
+0.9	4.16831-1	4.18493-1	4.17599-1	2.63639-0	6.99441-7	2.78827-7	1.41863-4	1.41874-4	1.41865-4
+1.0	4.16785-1	4.18335-1	4.17505-1	2.41801-0	6.98989-7	2.66219-7	1.41846-4	1.41857-4	1.41848-4
+1.1	4.16729-1	4.18256-1	4.17439-1	3.53712-4	6.98538-7	2.61077-7	1.41829-4	1.41840-4	1.41831-4
+1.2	4.16672-1	4.18109-1	4.17353-1	3.53842-4	6.98087-7	2.50496-7	1.41812-4	1.41822-4	1.41813-4
+1.3	4.16627-1	4.18008-1	4.17269-1	3.53972-4	6.97637-7	2.40081-7	1.41794-4	1.41805-4	1.41796-4
+1.4	4.16581-1	4.17895-1	4.17183-1	2.43522-1	6.45041-7	2.26574-7	1.41777-4	1.41787-4	1.41779-4
+1.5	4.16525-1	4.17759-1	4.17103-1	3.54232-4	6.18332-7	2.16409-7	1.41760-4	1.41769-4	1.41762-4
+1.6	4.16479-1	4.17646-1	4.17020-1	3.54362-4	5.91499-7	2.04750-7	1.41743-4	1.41751-4	1.41745-4
+1.7	4.16423-1	4.17533-1	4.16934-1	3.54492-4	5.64543-7	1.92222-7	1.41726-4	1.41733-4	1.41727-4
+1.8	4.16377-1	4.17386-1	4.16855-1	5.81153-0	5.37464-7	1.83155-7	1.41709-4	1.41716-4	1.41710-4
+1.9	4.16332-1	4.17284-1	4.16774-1	3.54752-4	5.10262-7	1.71480-7	1.41693-4	1.41698-4	1.41693-4
+2.0	4.16275-1	4.17159-1	4.16691-1	2.03817-0	4.55786-7	1.59088-7	1.41676-4	1.41680-4	1.41676-4
+2.1	4.16230-1	4.17046-1	4.16611-1	5.93188-0	4.28201-7	1.48342-7	1.41659-4	1.41663-4	1.41659-4
+2.2	4.16173-1	4.16910-1	4.16525-1	2.87727-0	4.00493-7	1.35071-7	1.41642-4	1.41645-4	1.41642-4
+2.3	4.16128-1	4.16808-1	4.16441-1	3.55273-4	3.72665-7	1.22637-7	1.41625-4	1.41628-4	1.41626-4
+2.4	4.16082-1	4.16695-1	4.16368-1	3.55403-4	3.44715-7	1.12948-7	1.41608-4	1.41611-4	1.41609-4
+2.5	4.16025-1	4.16570-1	4.16280-1	2.87726-0	2.88642-7	9.88080-8	1.41592-4	1.41593-4	1.41592-4
+2.6	4.15980-1	4.16457-1	4.16198-1	5.81668-0	2.60313-7	8.63478-8	1.41575-4	1.41576-4	1.41575-4
+2.7	4.15935-1	4.16332-1	4.16121-1	3.59267-0	2.31864-7	7.45967-8	1.41558-4	1.41559-4	1.41558-4
+2.8	4.15878-1	4.16230-1	4.16035-1	3.55924-4	2.03296-7	6.24520-8	1.41541-4	1.41542-4	1.41541-4
+2.9	4.15832-1	4.16139-1	4.15966-1	3.25843-0	1.74609-7	5.26473-8	1.41525-4	1.41525-4	1.41525-4
+3.0	4.15787-1	4.16014-1	4.15881-1	2.94873-0	1.16954-7	3.83979-8	1.41508-4	1.41508-4	1.41508-4
+3.1	4.15741-1	4.15912-1	4.15803-1	3.56315-4	8.78926-8	2.64618-8	1.41491-4	1.41492-4	1.41491-4
+3.2	4.15685-1	4.15787-1	4.15728-1	2.96461-0	5.87128-8	1.64697-8	1.41475-4	1.41475-4	1.41475-4
+3.3	4.15639-1	4.15696-1	4.15643-1	1.82012-0	2.94151-8	2.20613-9	1.41458-4	1.41458-4	1.41458-4
+3.4	4.15582-1	4.15582-1	4.15582-1	4.54013-0	4.54013-0	4.54013-0	1.41442-4	1.41442-4	1.41442-4
Total	4.15582-1	4.20969-1	4.18022-1	1.39959-1	8.10834-7	2.44702-7	1.41442-4	1.42626-4	1.42027-4

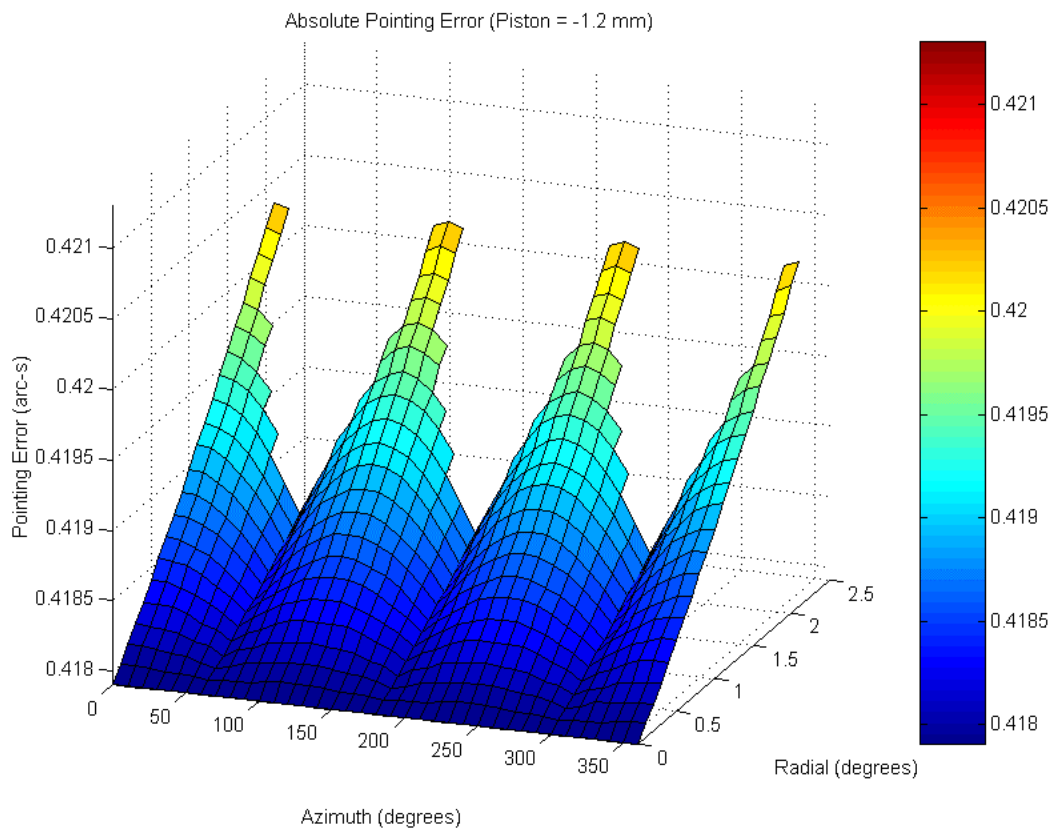
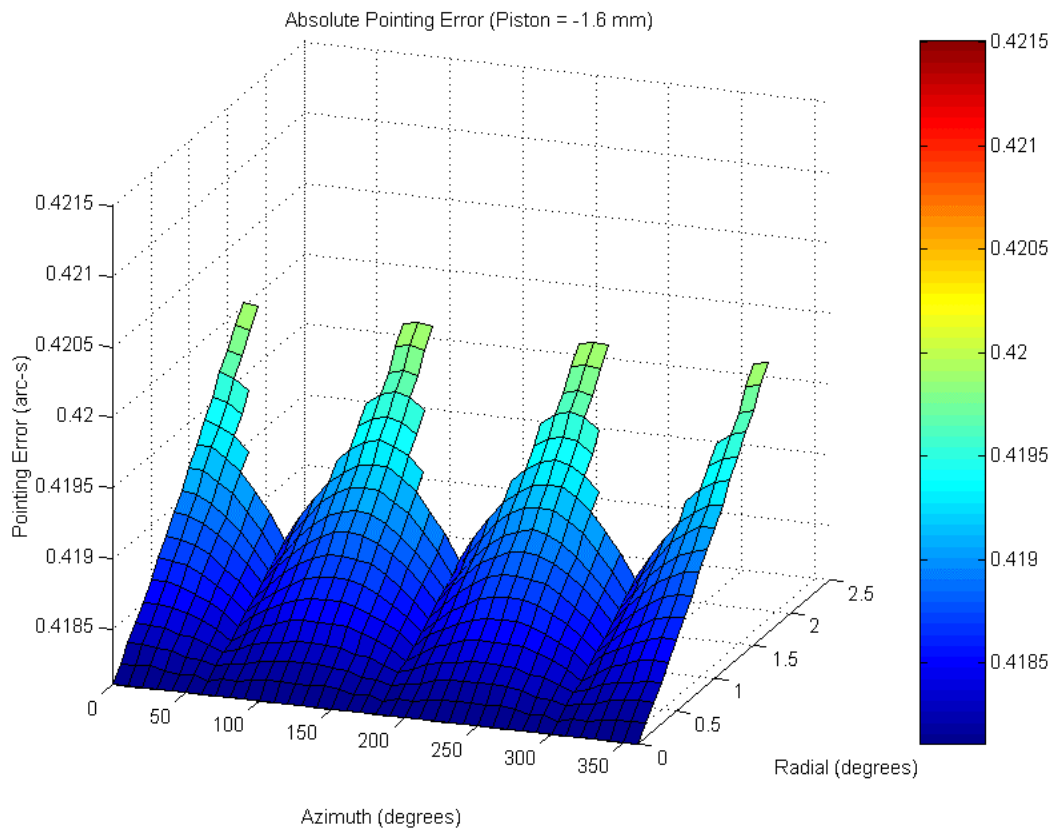
ABSOLUTE POINTING ERROR MAPS OF THE NOMINAL HEXAPOD DESIGN

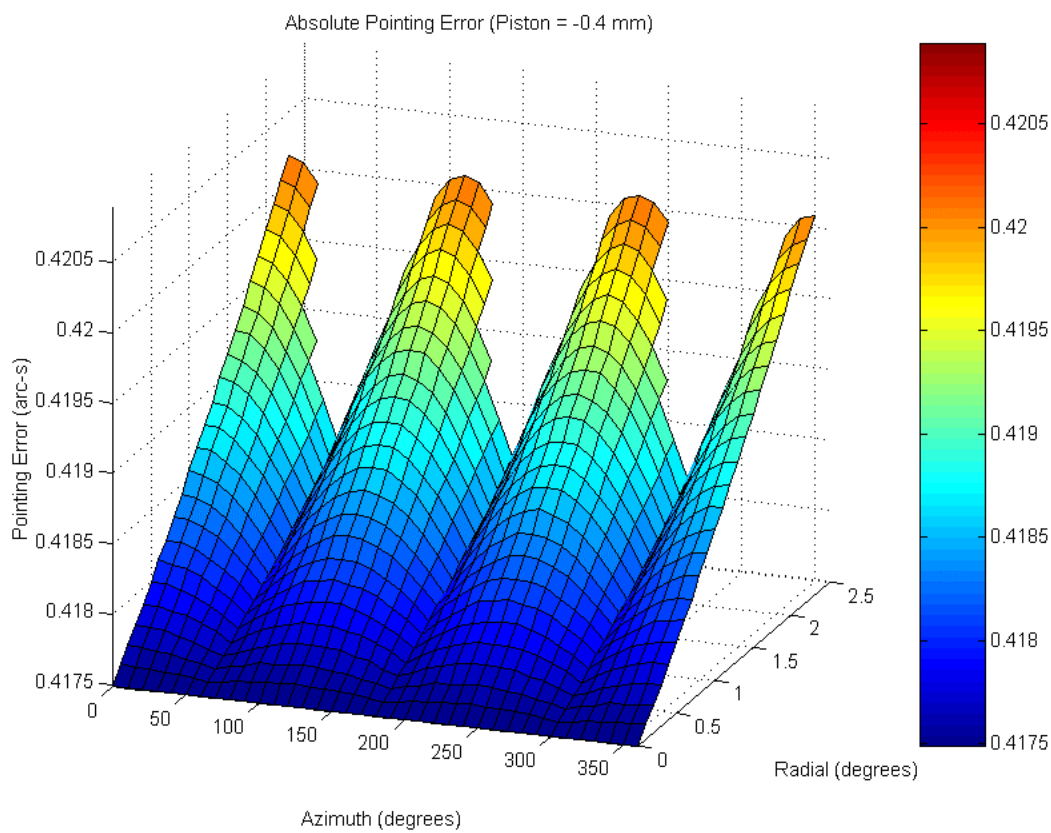
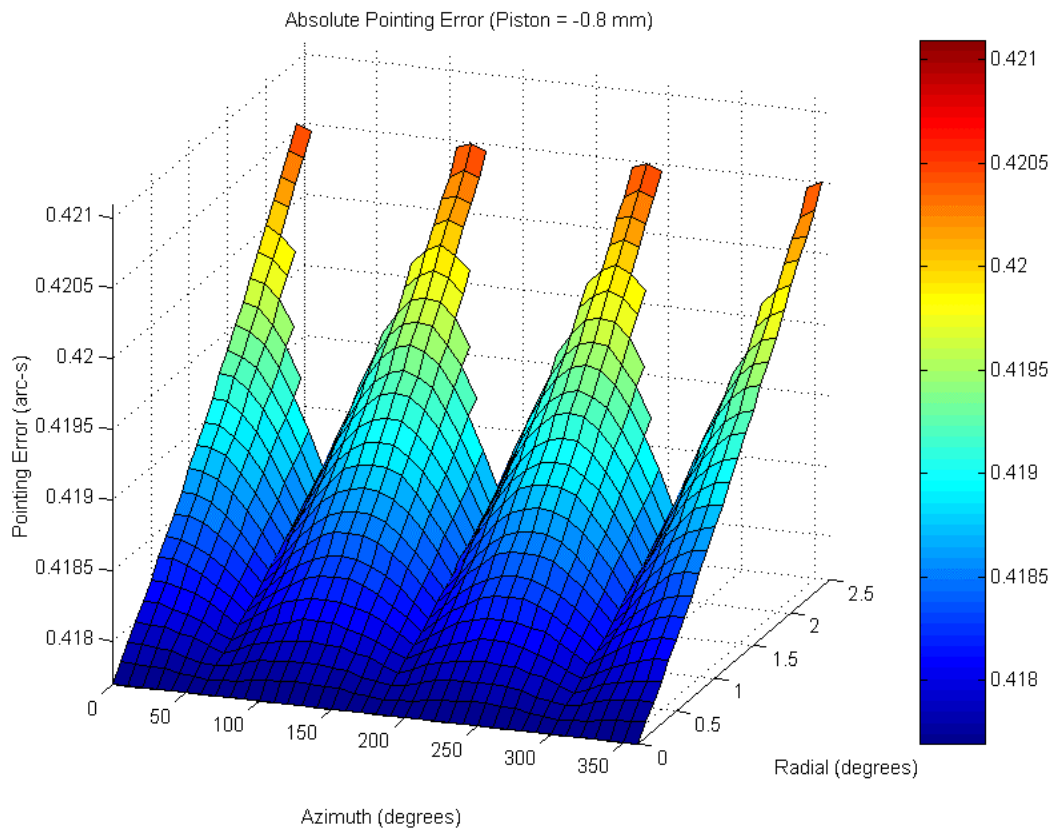
The following series of plots show the pointing accuracy over the entire operational range of the nominal hexapod design. Each plot corresponds to a single value of piston at which the hexapod was then moved in pure rotations of radial and azimuth pointing to complete the plot. All plots in the series are presented with the same total range for the pointing error (along the vertical axis) so that all data is uniformly scaled from one plot to the next. However, it should be noted that the minimum and maximum range extents of the vertical axis tend to shift downward in each successive plot. This shift reflects the fact that the pointing error varies indirectly as a function of the hexapod's overall height. Nevertheless, the plots show an even stronger dependence on radial pointing angle. In other words, pointing error tends to increase most quickly with radial pointing angle, which has a larger range when the hexapod is positioned with a mid-range piston value. It should be noted that the hexapod can produce positions with piston components between -3.4 and $+3.4$ mm. At these extremes the radial pointing range is zero since the entire range of each actuator has already been used to produce the piston. This is why the maximum pointing error occurs when the hexapod has a piston value of -1.3 mm. It should also be noted that there is a moderate dependence of the pointing accuracy on the azimuth-pointing angle of the hexapod, and this dependence is responsible for the three lobed, repeating pattern that is readily seen in all the plots.

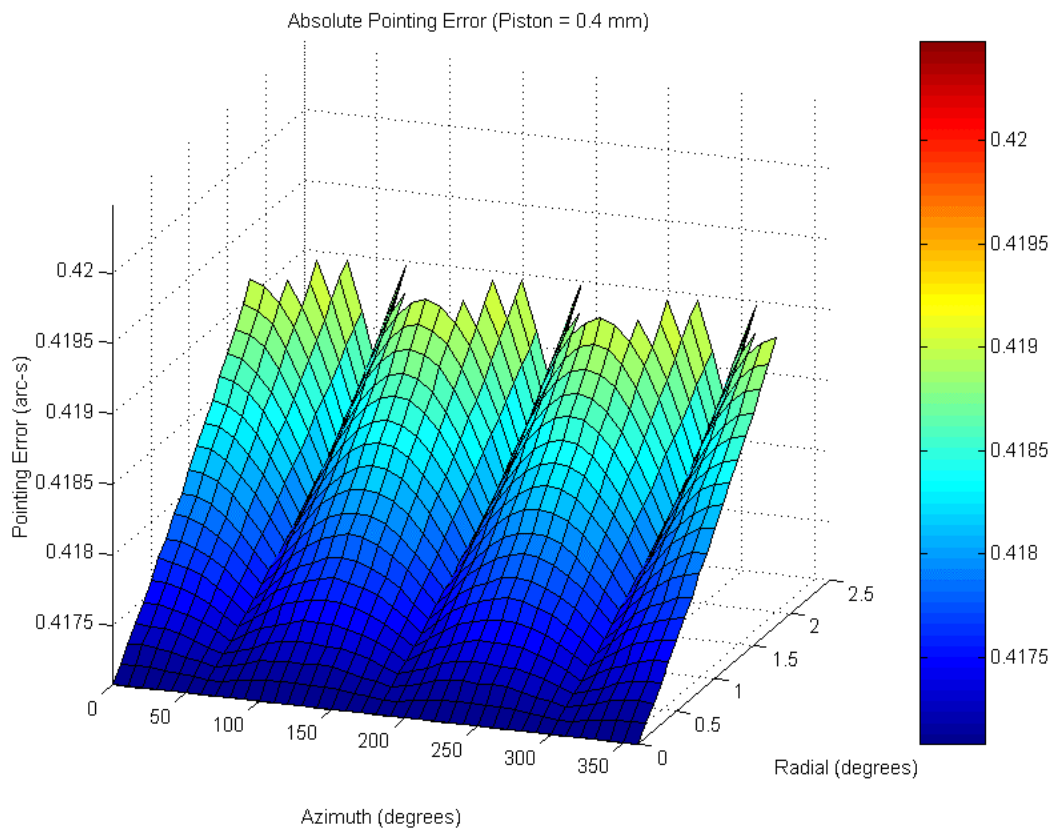
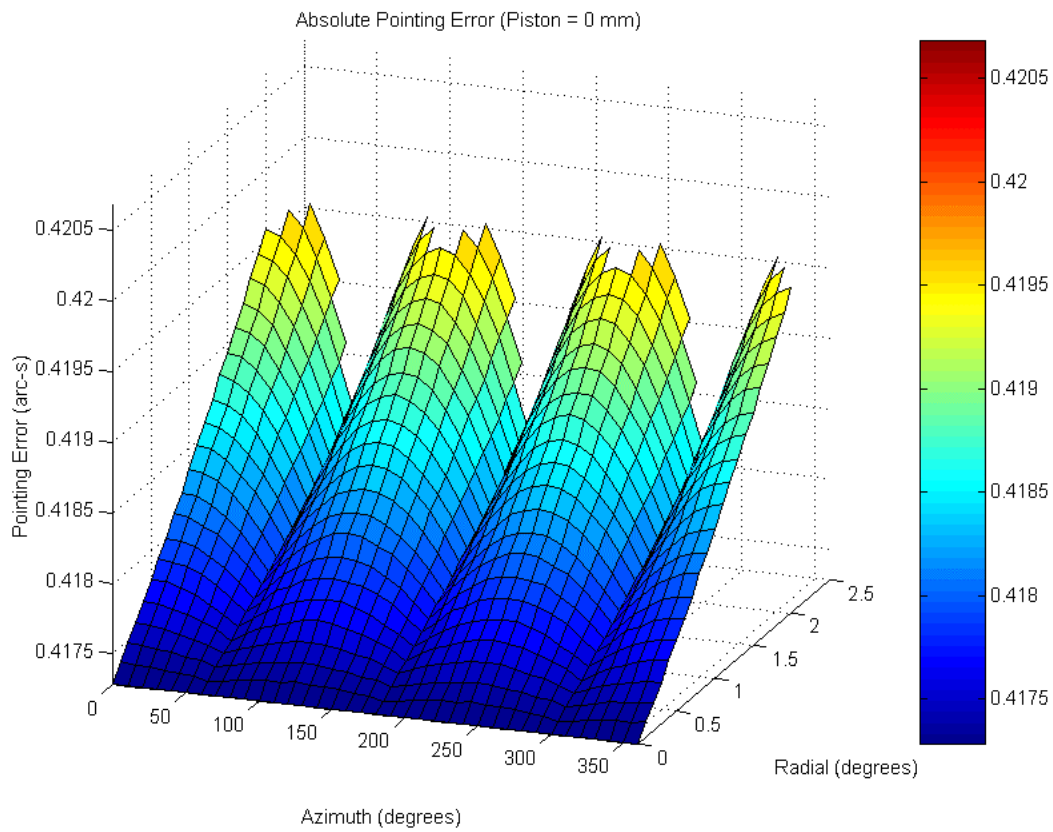
NOTE: These plots represent only a subset of the data presented in the previous table which displays hexapod position error at successive increments of 0.1 mm over the range from -3.4 to $+3.4$ mm in piston. The following plots display data at increments of 0.4 mm over the range from -2.8 to $+2.8$ mm.

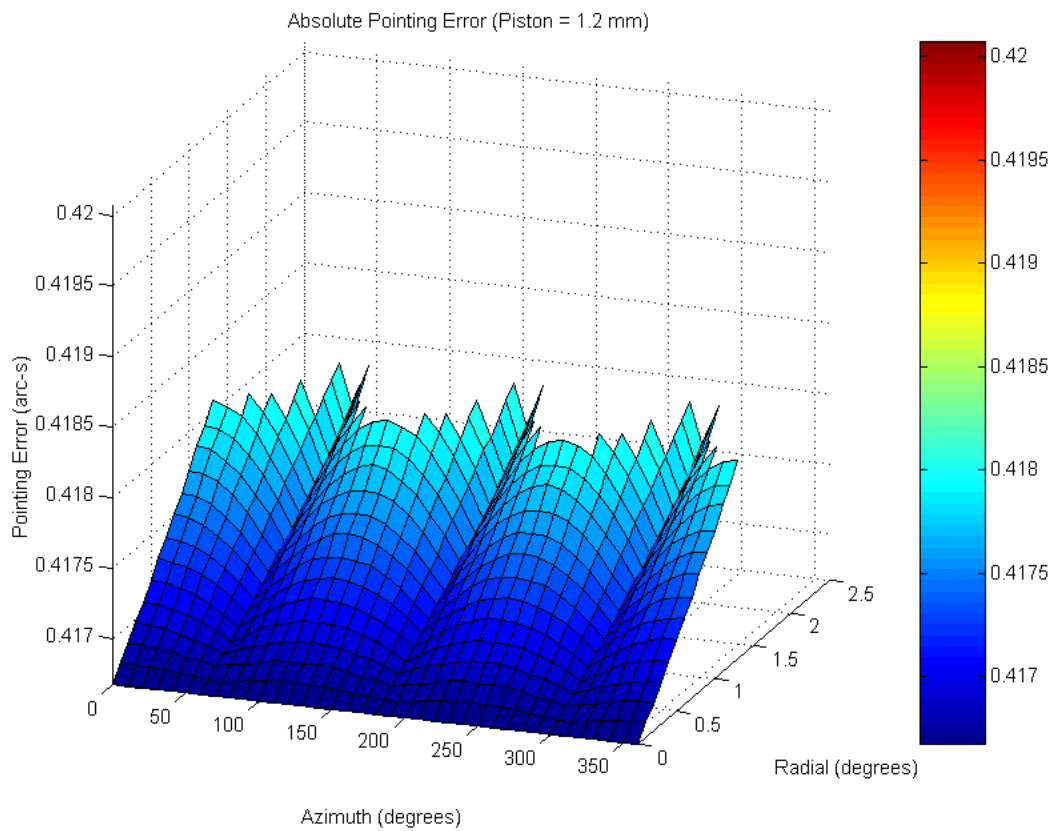
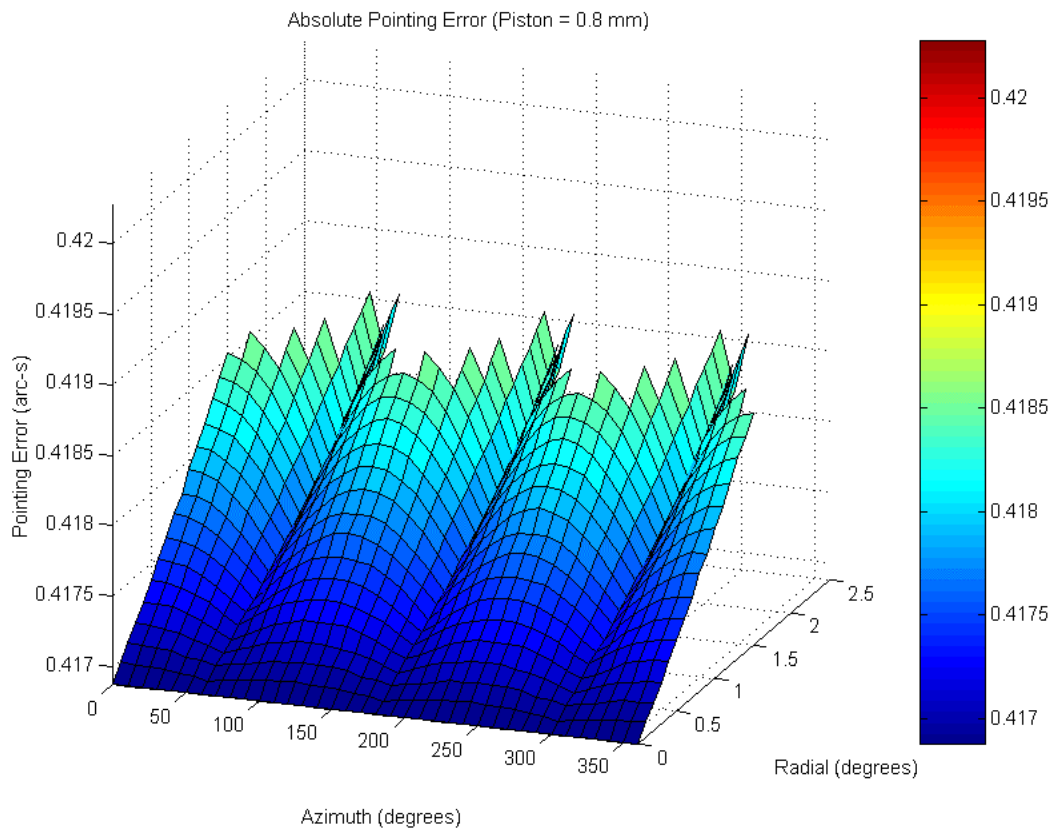


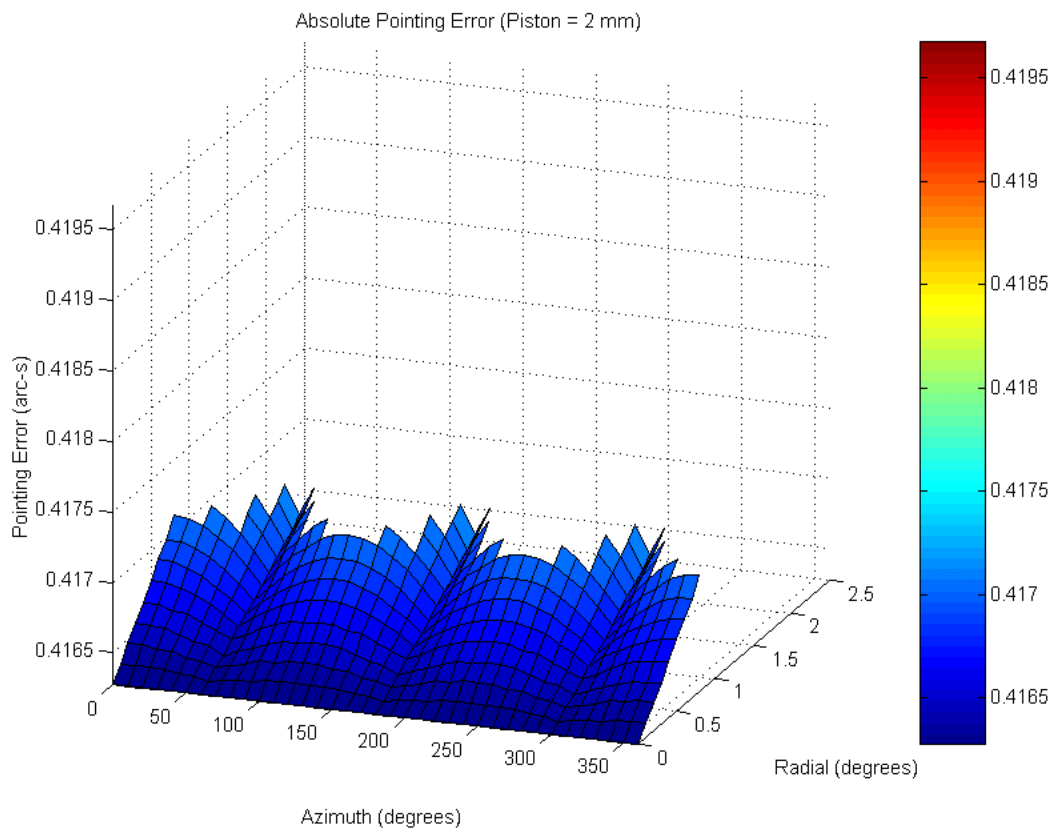
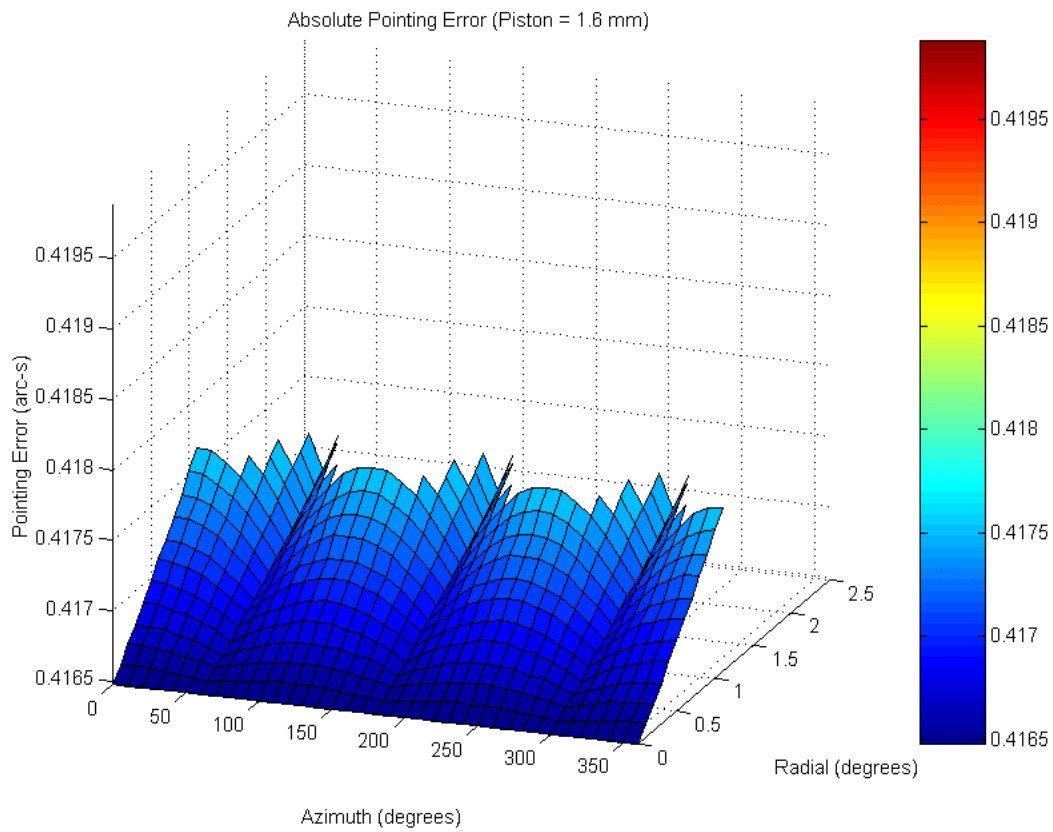


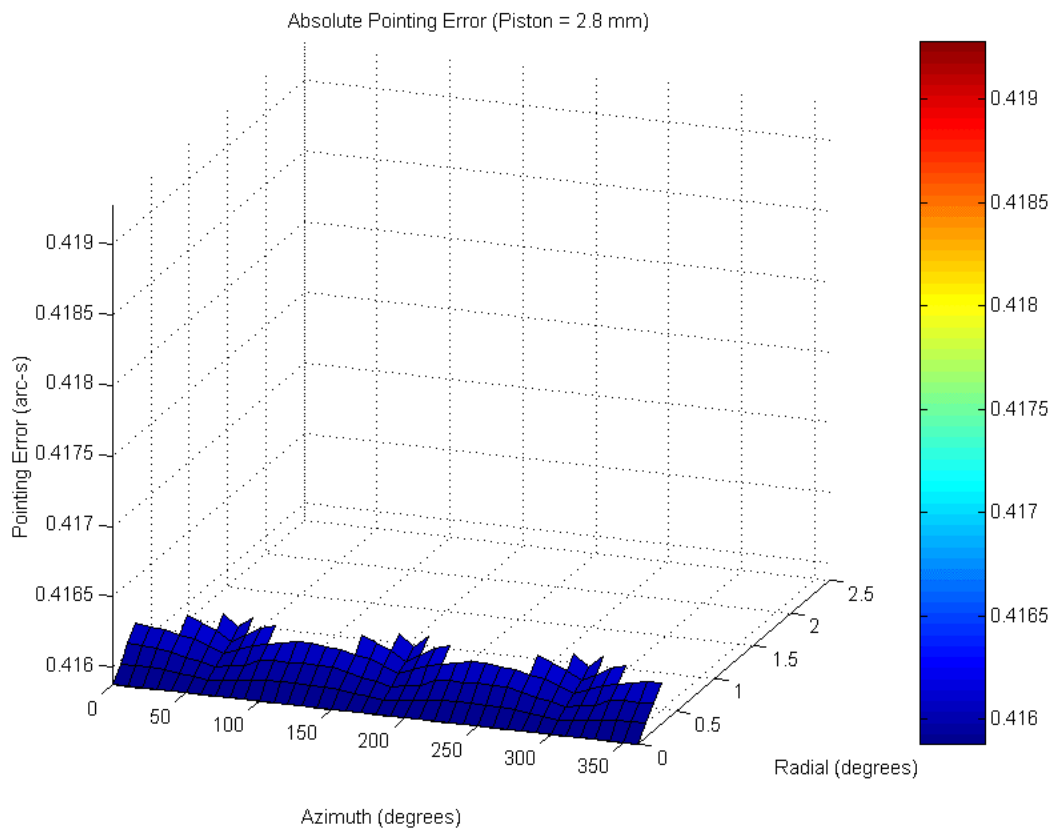
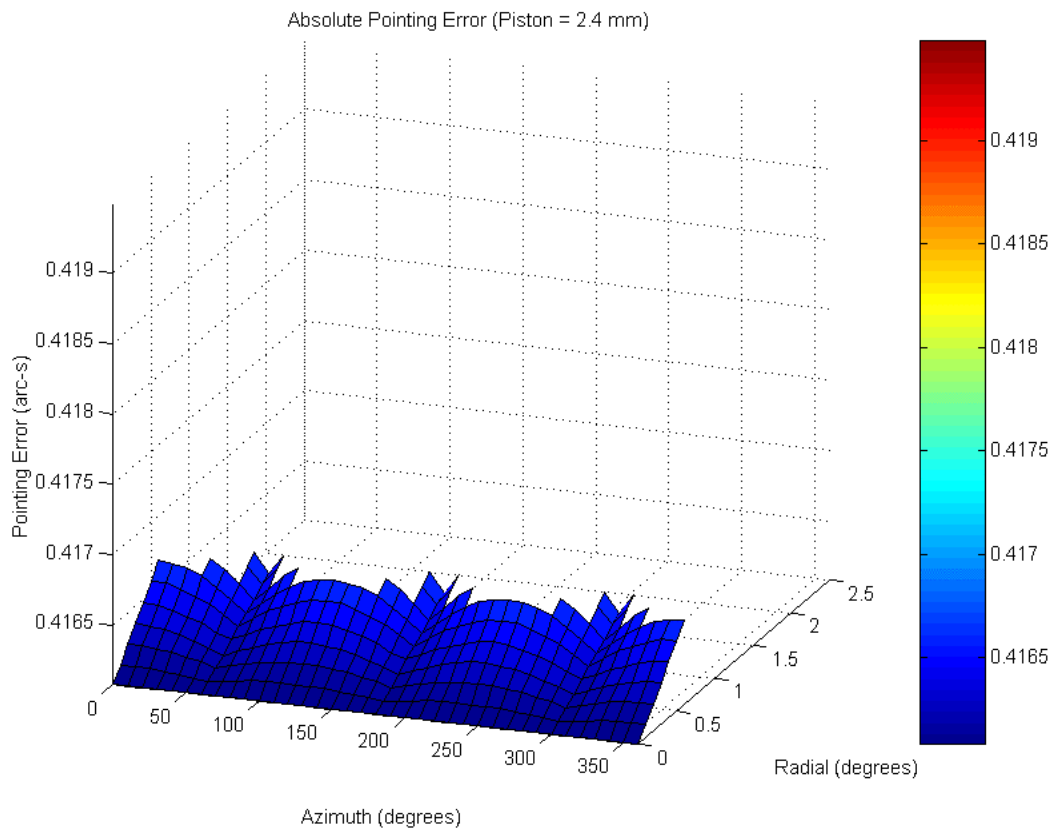








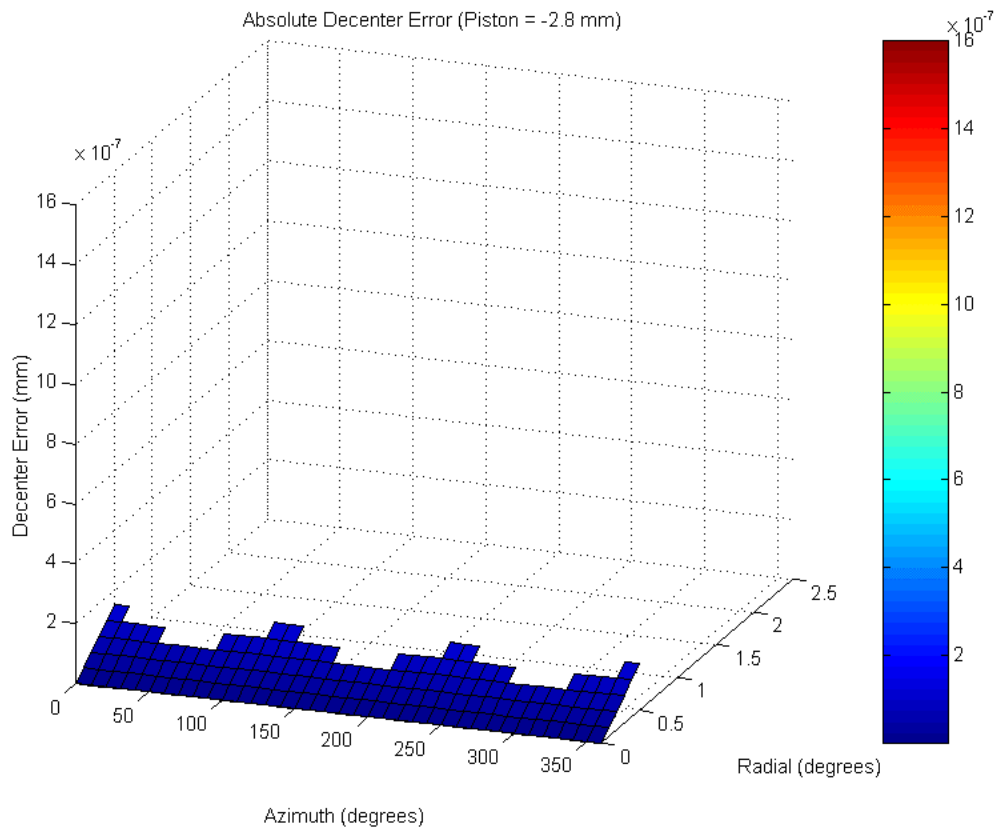


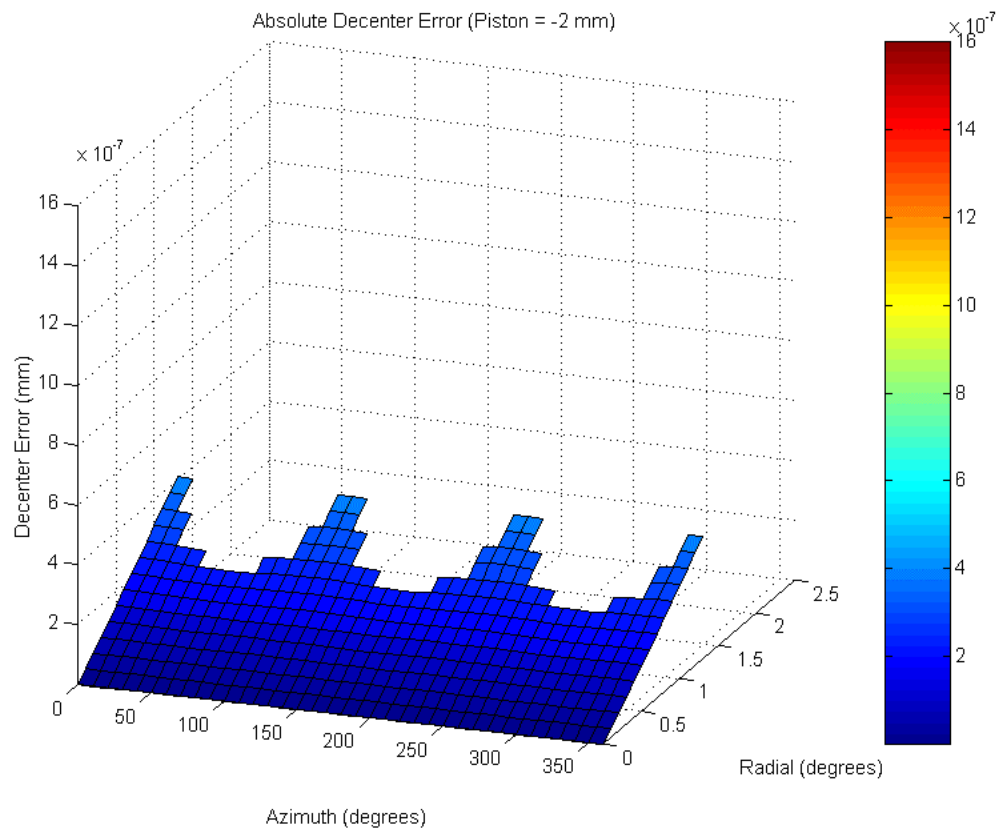
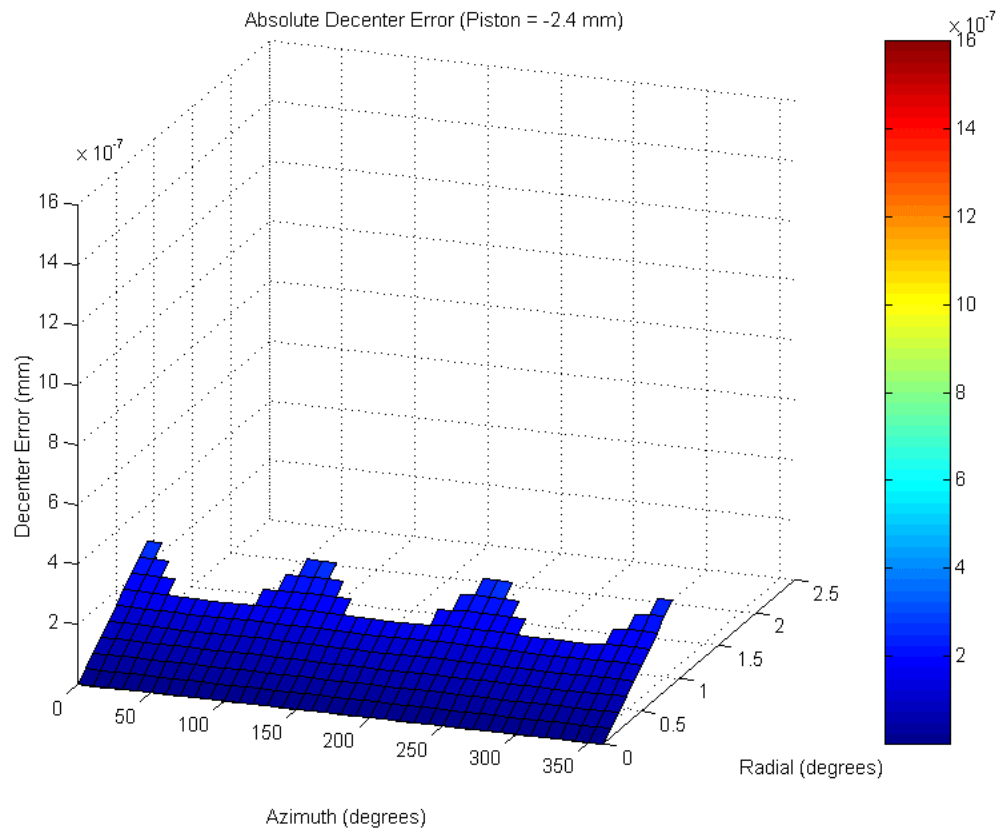


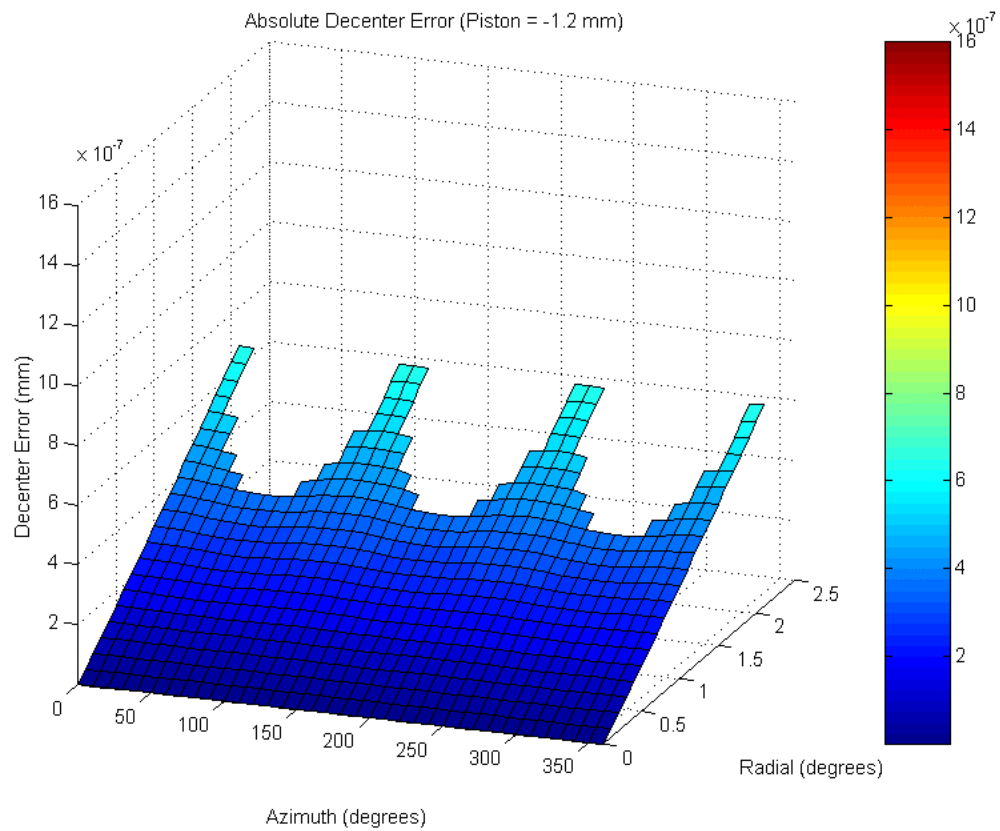
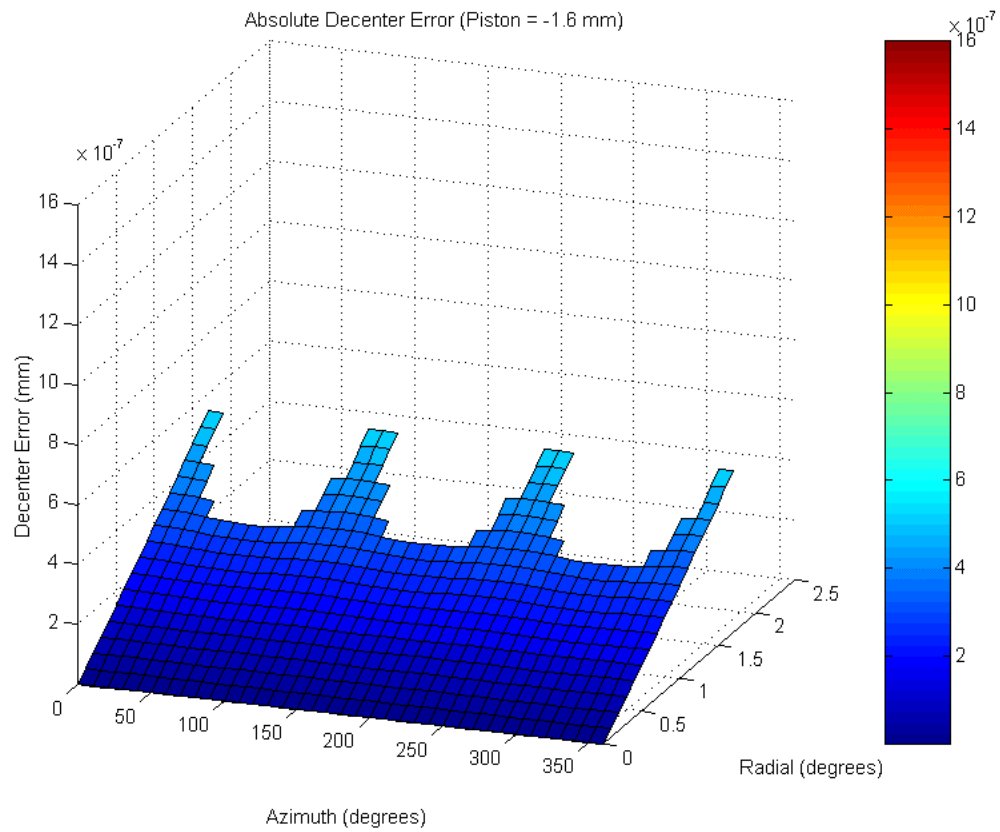
ABSOLUTE DECENTER ERROR MAPS OF THE NOMINAL HEXAPOD DESIGN

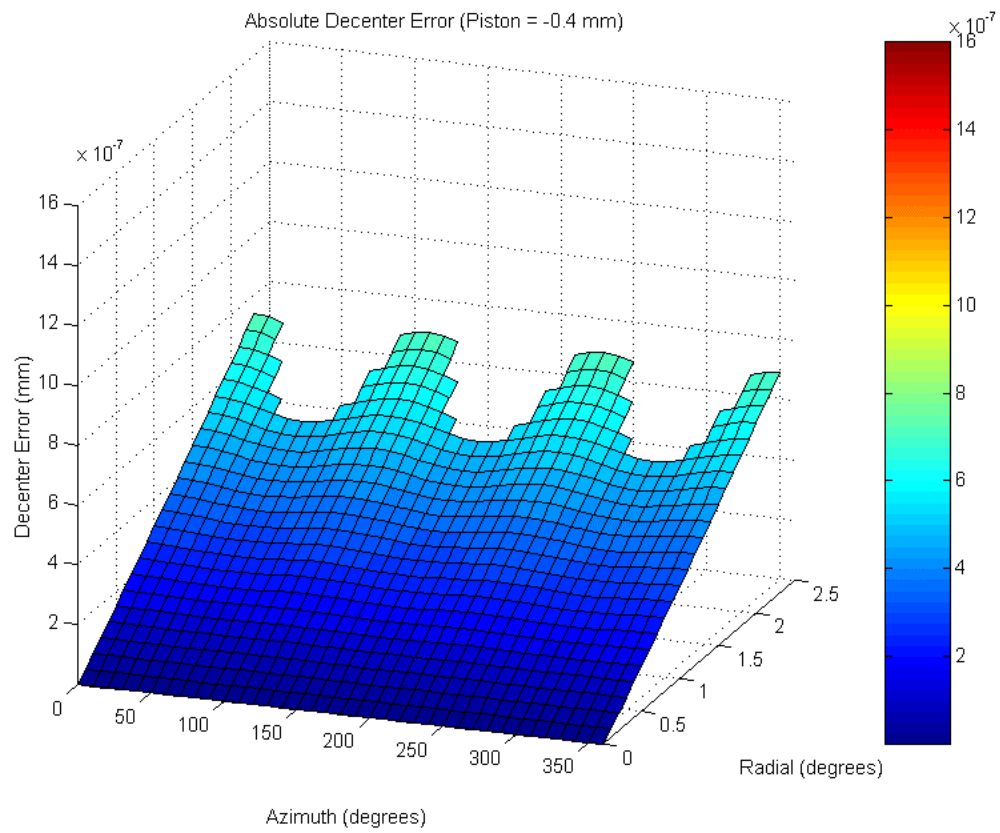
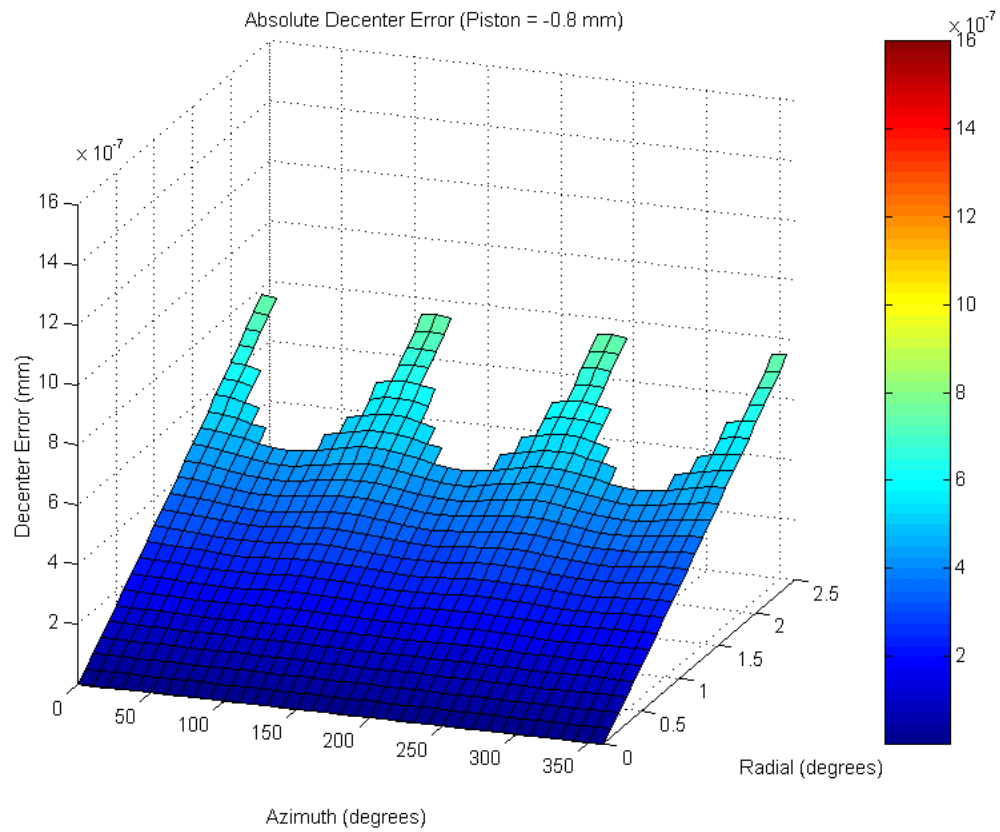
The following series of plots show the decenter error over the entire operational range of the nominal hexapod design. Each plot corresponds to a single value of piston at which the hexapod was then moved in pure rotations of radial and azimuth pointing to complete the plot. All plots in the series are presented with the same total range for the decenter error (along the vertical axis) so that all data is uniformly scaled from one plot to the next. It should be noted that the decenter error is almost entirely a function of the radial pointing angle of the hexapod.

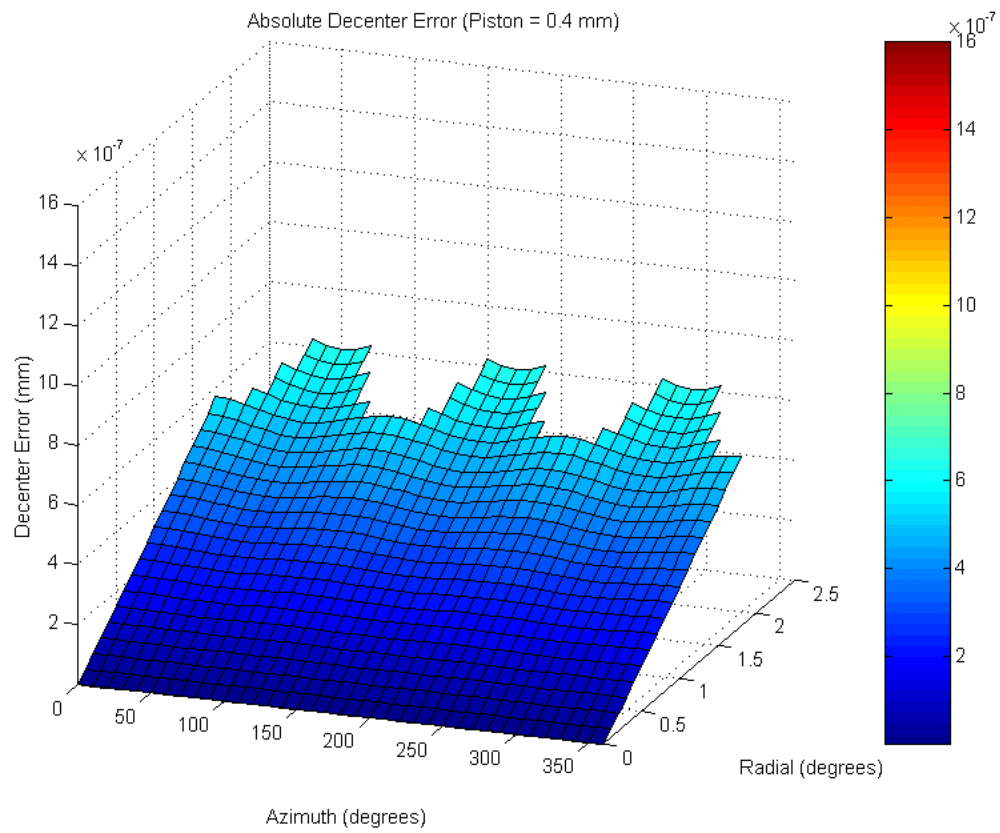
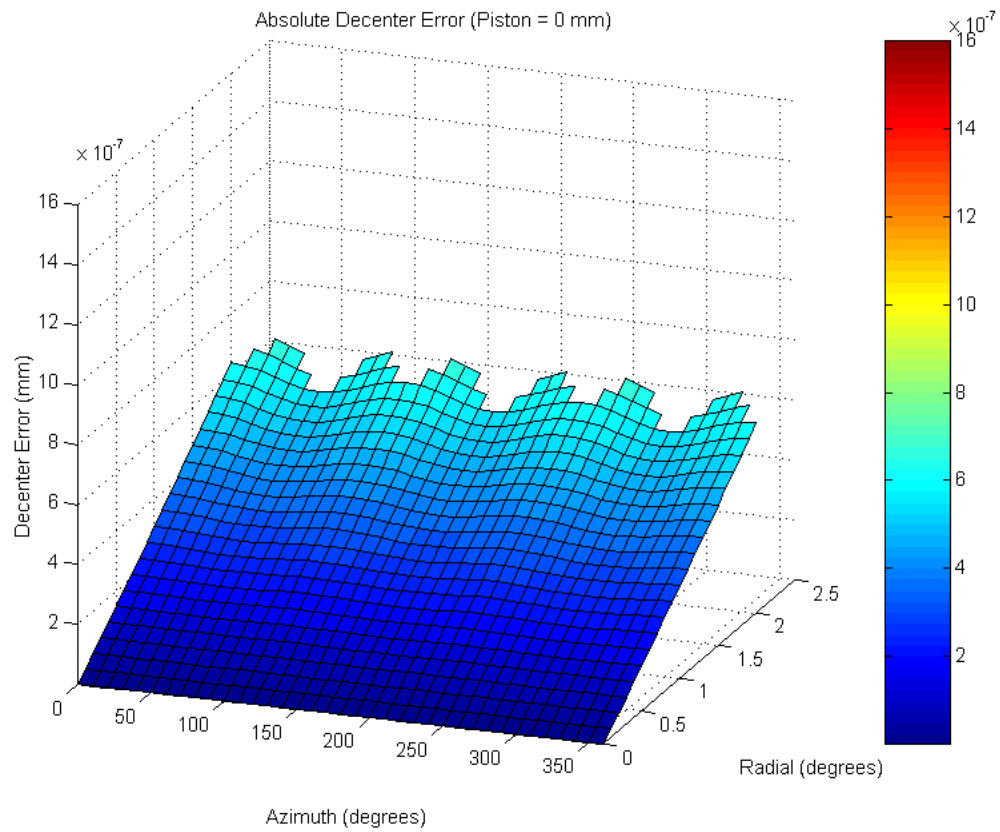
NOTE: These plots represent only a subset of the data presented in the previous table which displays hexapod position error at successive increments of 0.1 mm over the range from -3.4 to $+3.4$ mm in piston. The following plots display data at increments of 0.4 mm over the range from -2.8 to $+2.8$ mm.

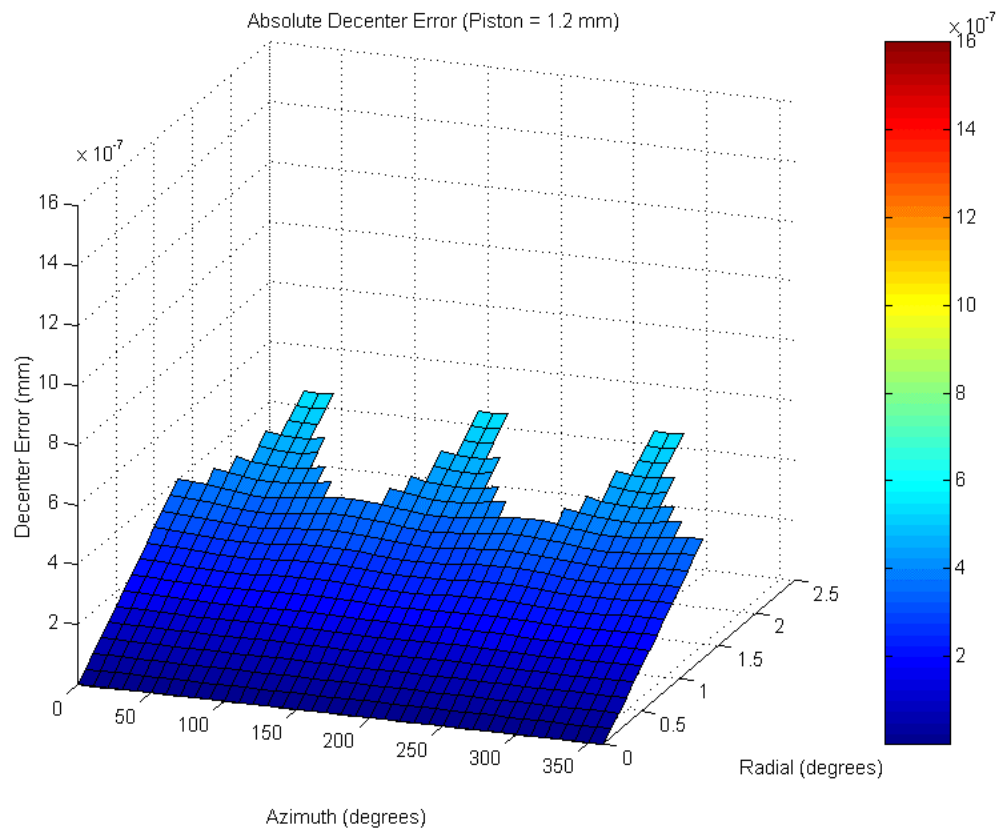
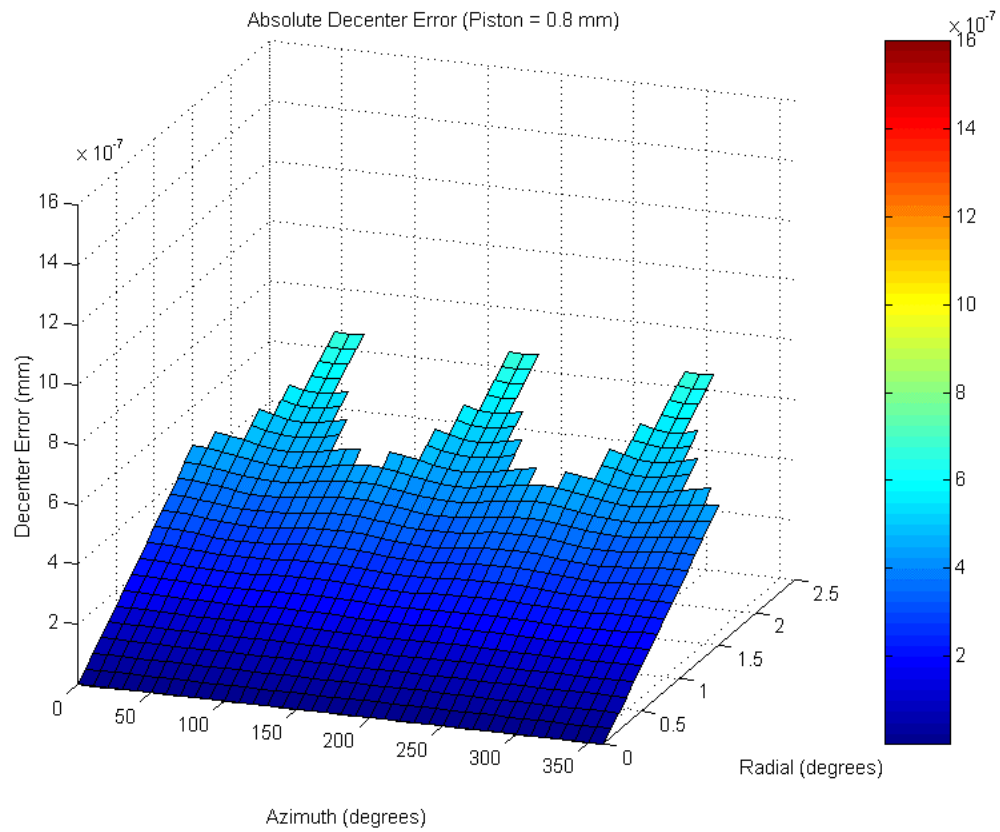


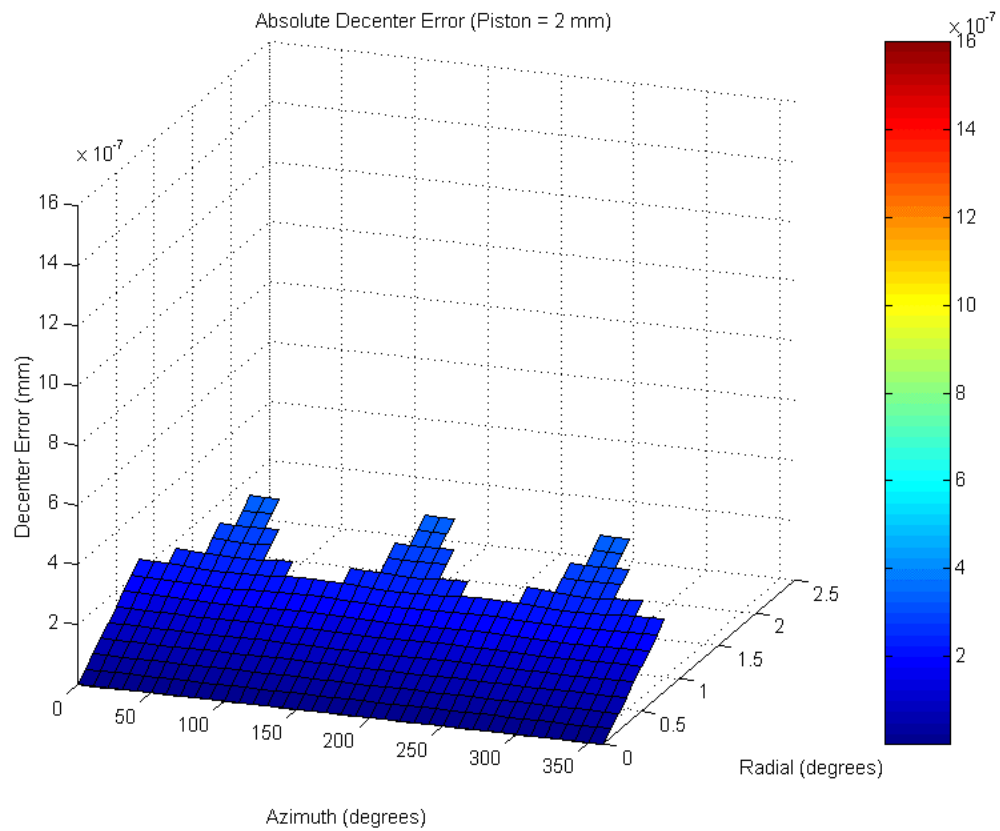
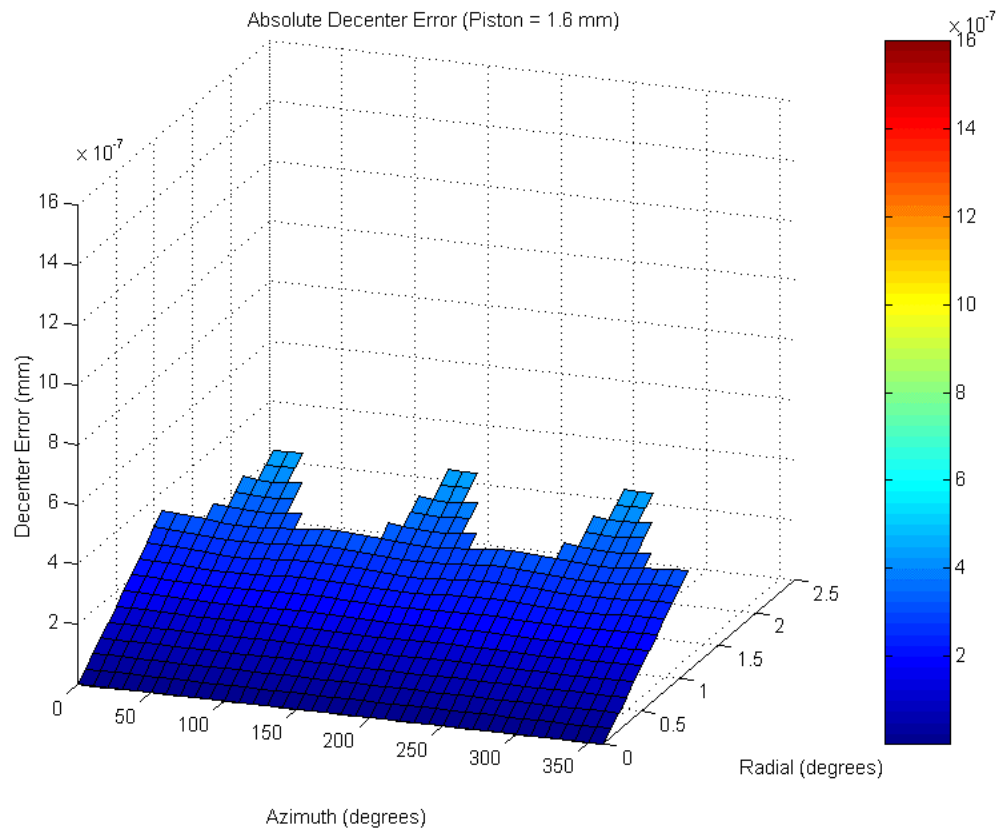


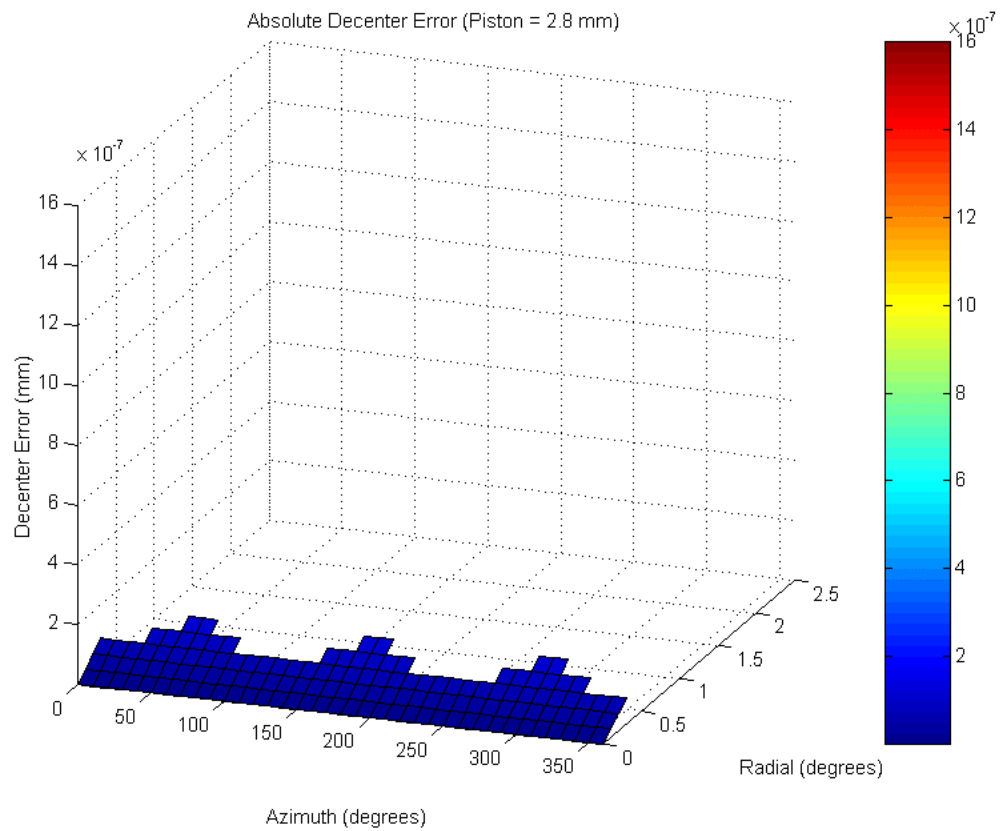
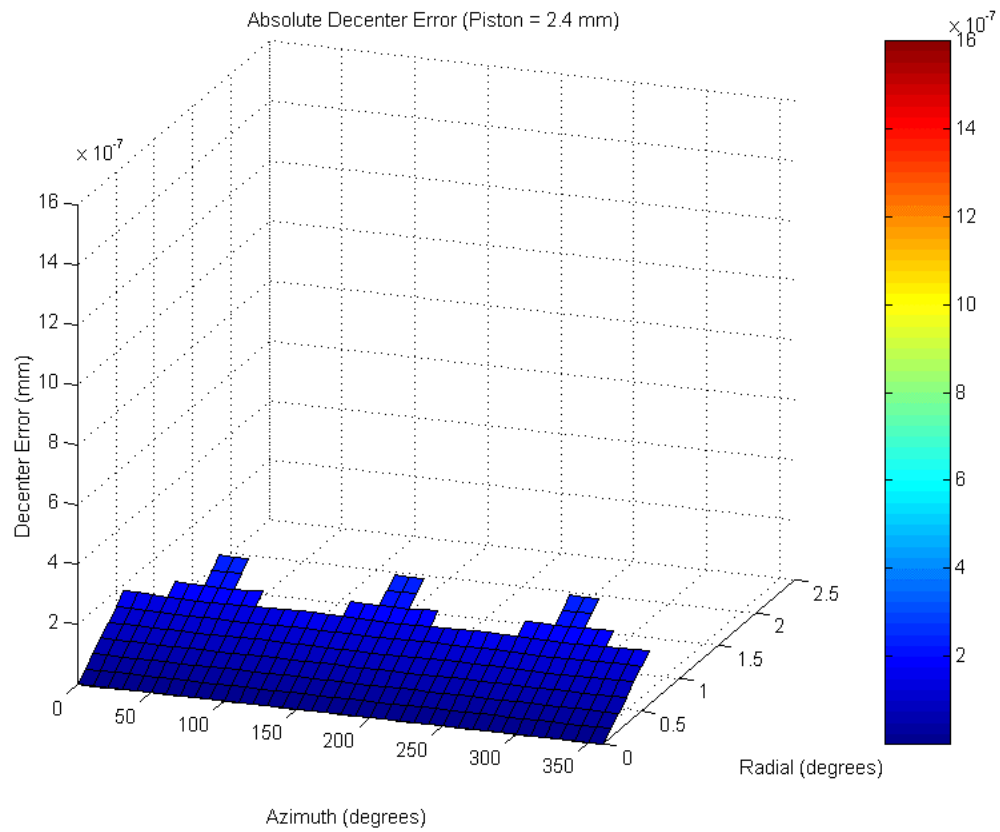










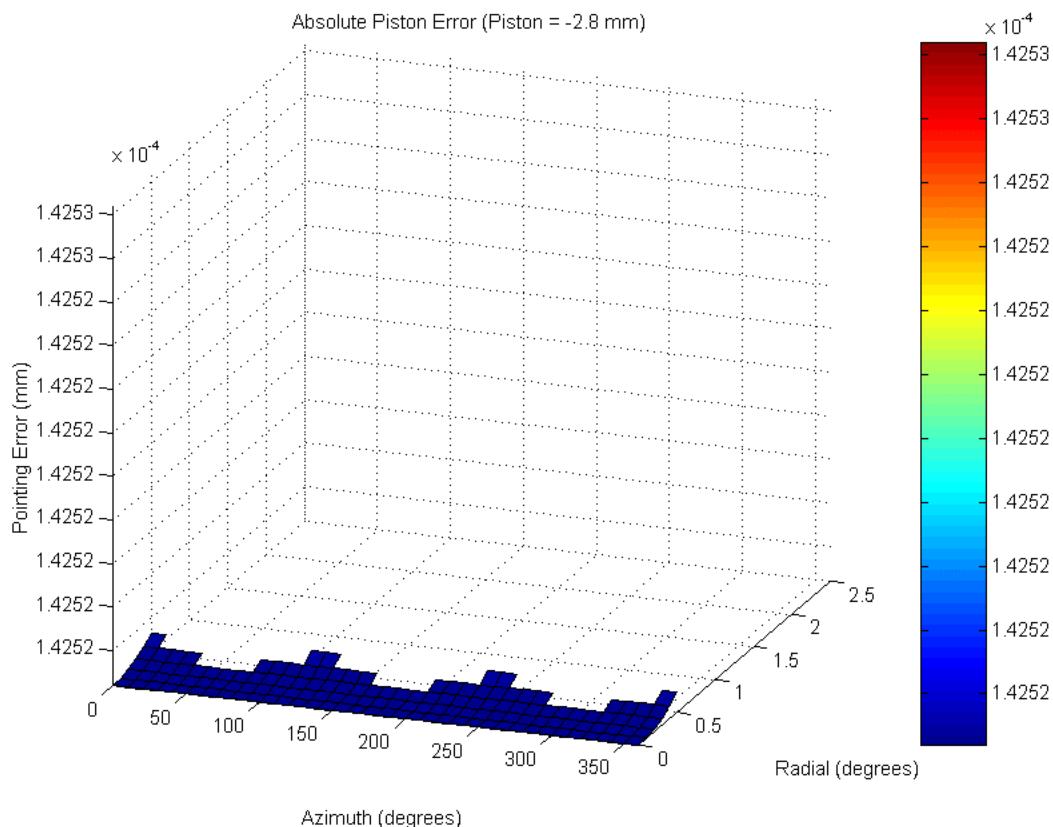


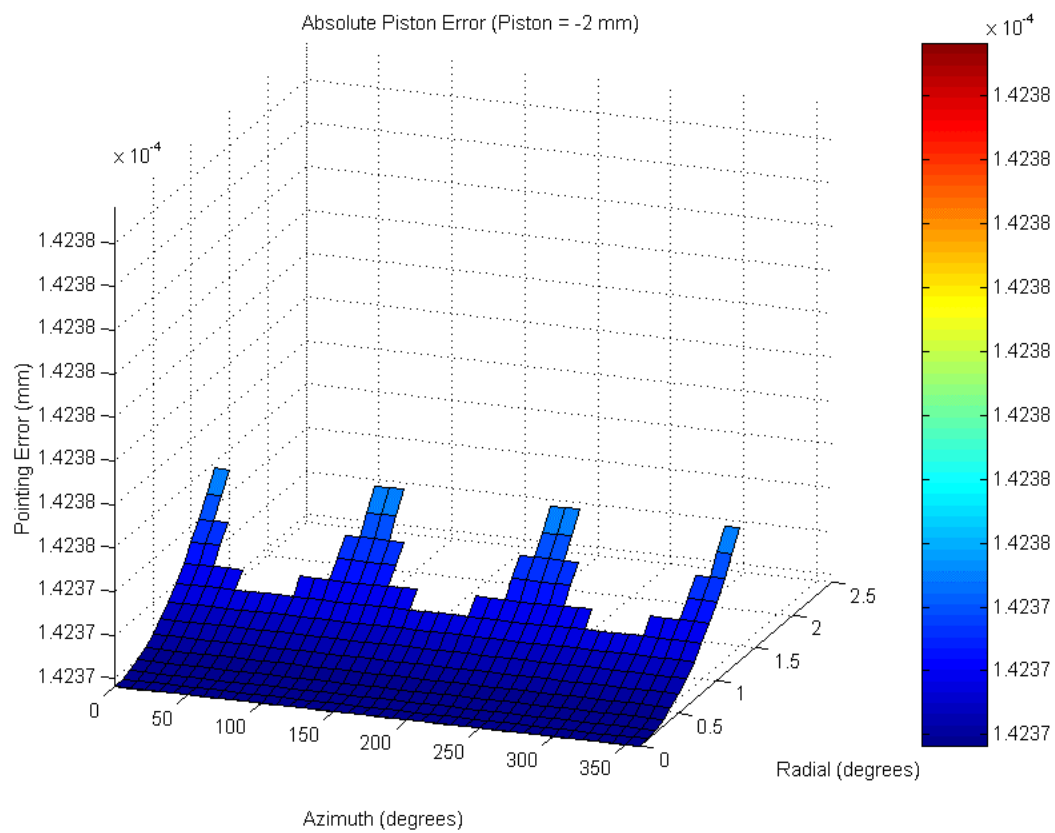
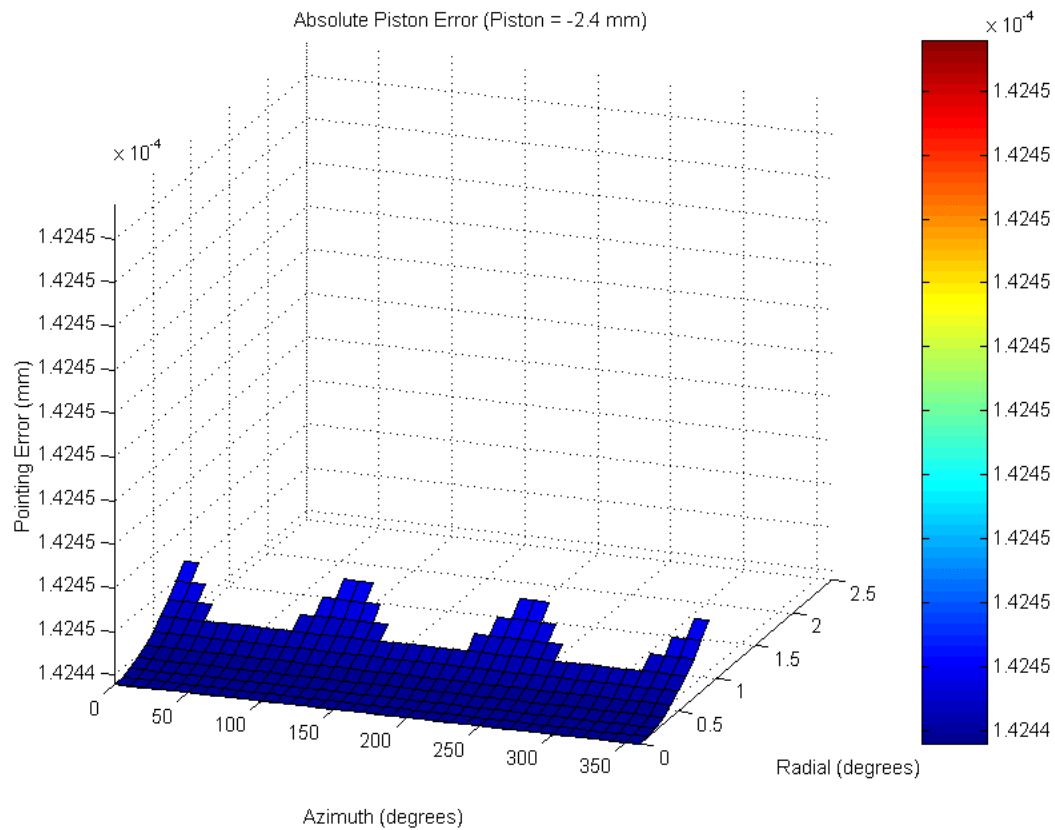
ABSOLUTE PISTON ERROR MAPS OF THE NOMINAL HEXAPOD DESIGN

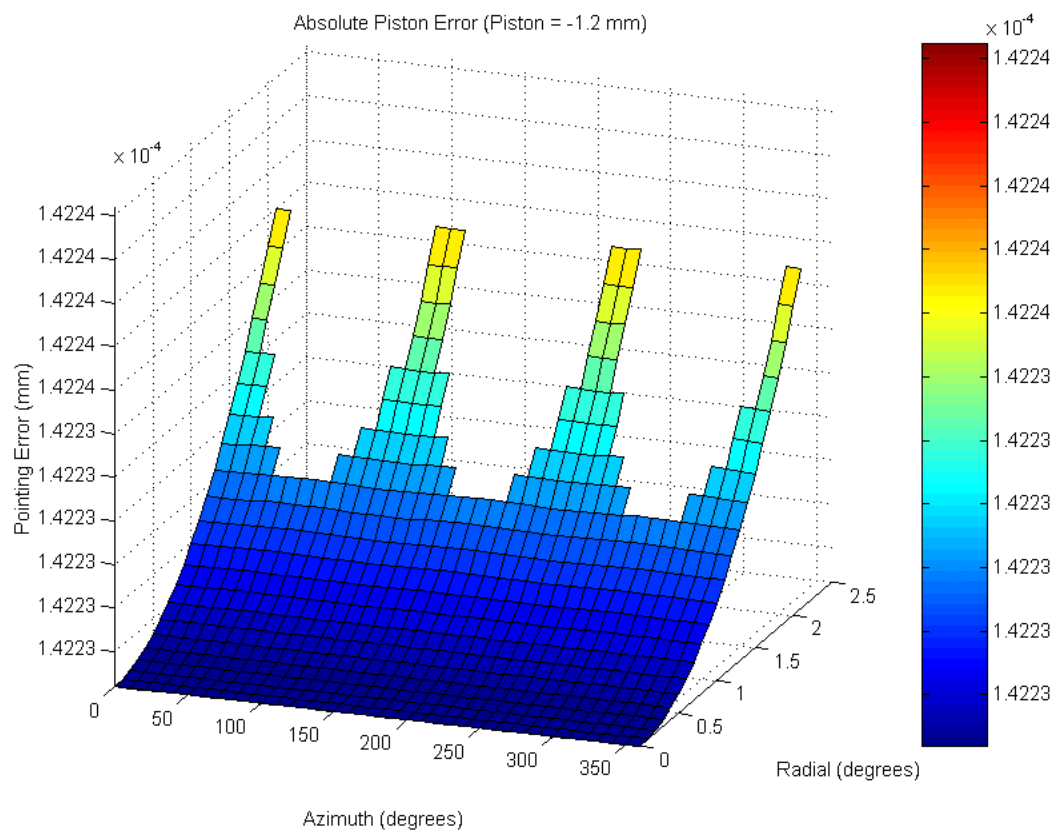
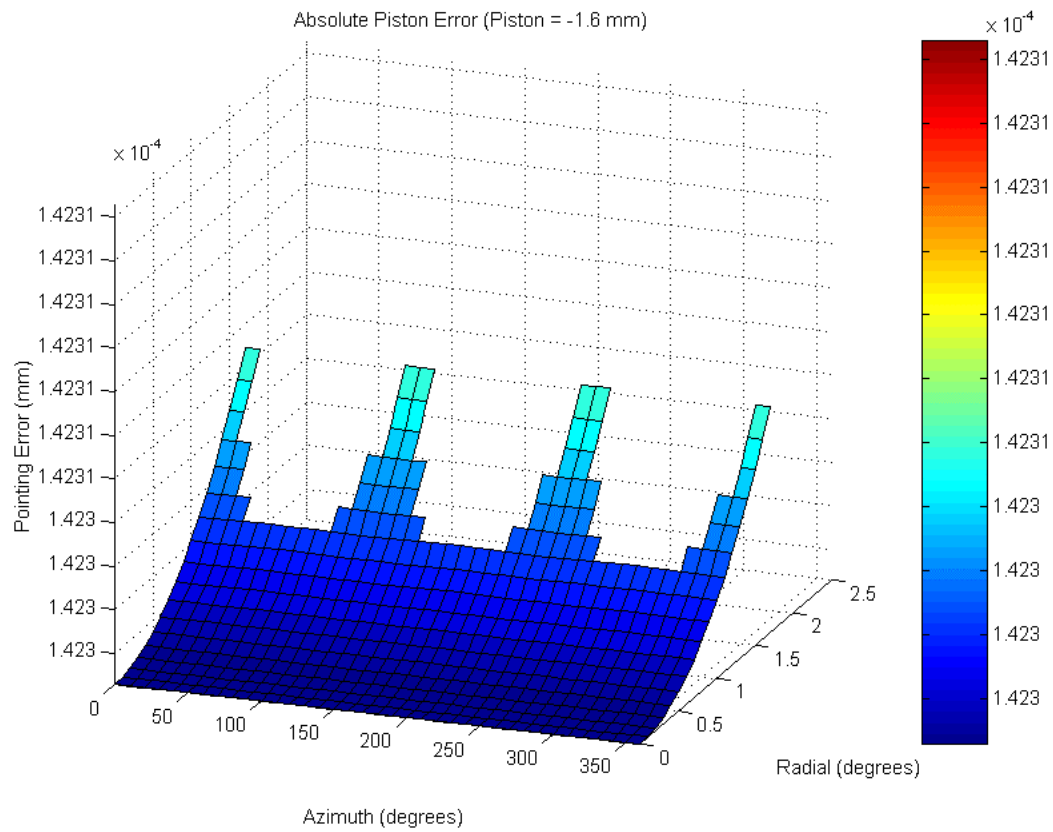
Of all the three component error sources that were evaluated, the piston error is the easiest to understand. This is because it's almost entirely a function of the hexapod's overall height. When reviewing single plots of the piston error, this is not readily apparent because each plot shows a definite increase in piston error as a function of radial pointing angle. However, it should be noted that the range of the piston error on a single plot is much smaller than the total range of piston error over all the plots combined. Since each successive plot represents an increase in the overall height of the hexapod, the predominant factor in determining the absolute piston error is the height. For the sake of completeness, it should also be noted that there is an extremely small dependence of the piston error on the azimuth-pointing angle of the hexapod as well. This shows up as a slight variation (or waviness) in the individual plots and is most noticeable in the plot corresponding to 0 mm of piston.

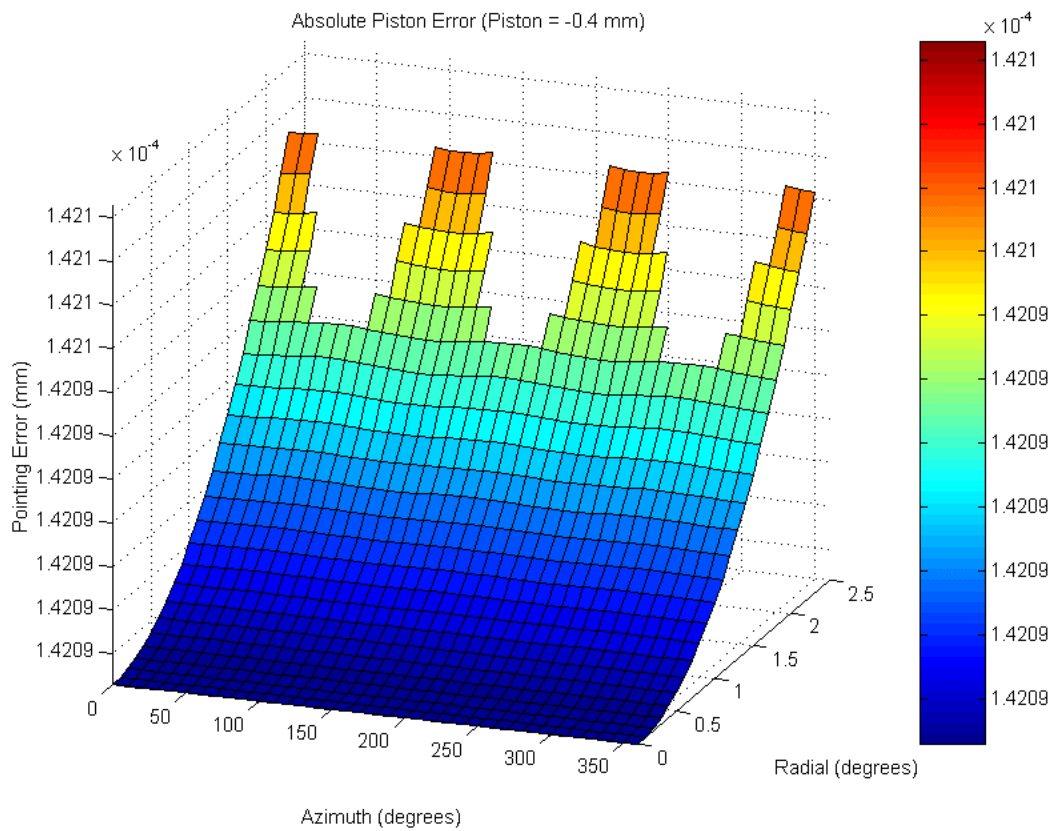
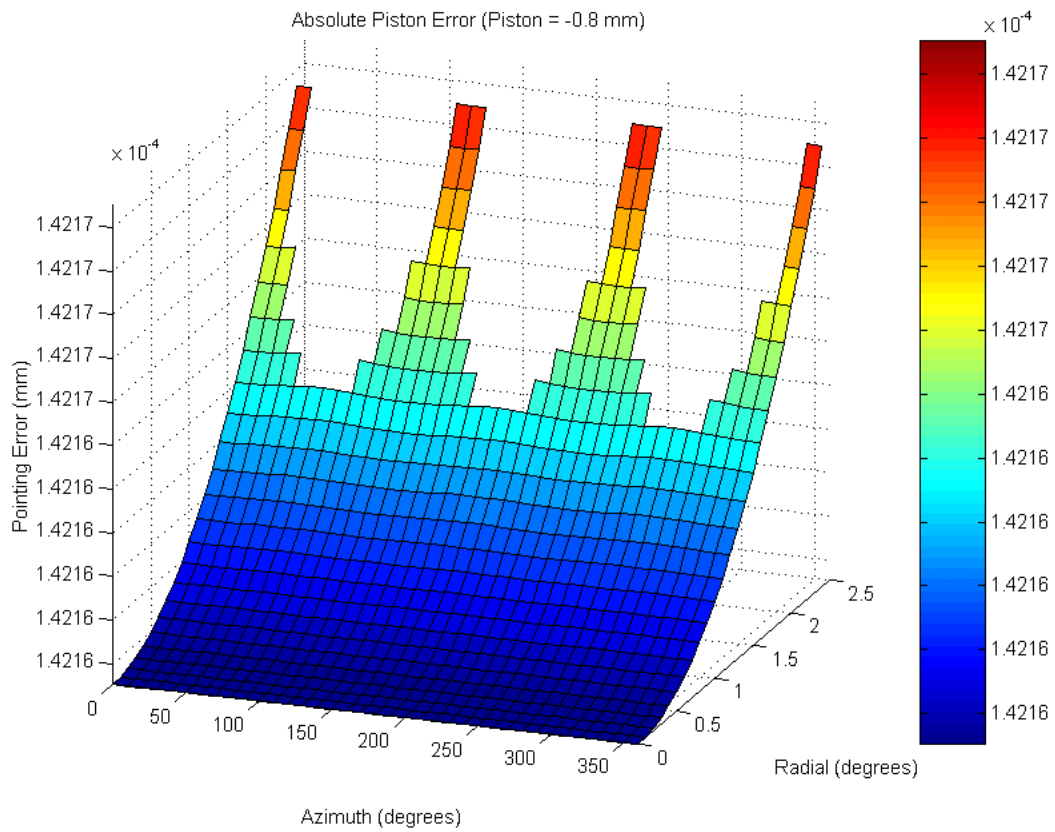
The following series of plots show the piston accuracy over the entire operational range of the nominal hexapod design. Each plot corresponds to a single value of piston at which the hexapod was then moved in pure rotations of radial and azimuth pointing to complete the plot. All plots in the series are presented with the same total range for the pointing error (along the vertical axis) so that all data is uniformly scaled from one plot to the next. However, it should be noted that the minimum and maximum range extents of the vertical axis tend to shift upward in each successive plot. This shift reflects the fact that the pointing error varies directly as a function of the hexapod's overall height (as previously mentioned).

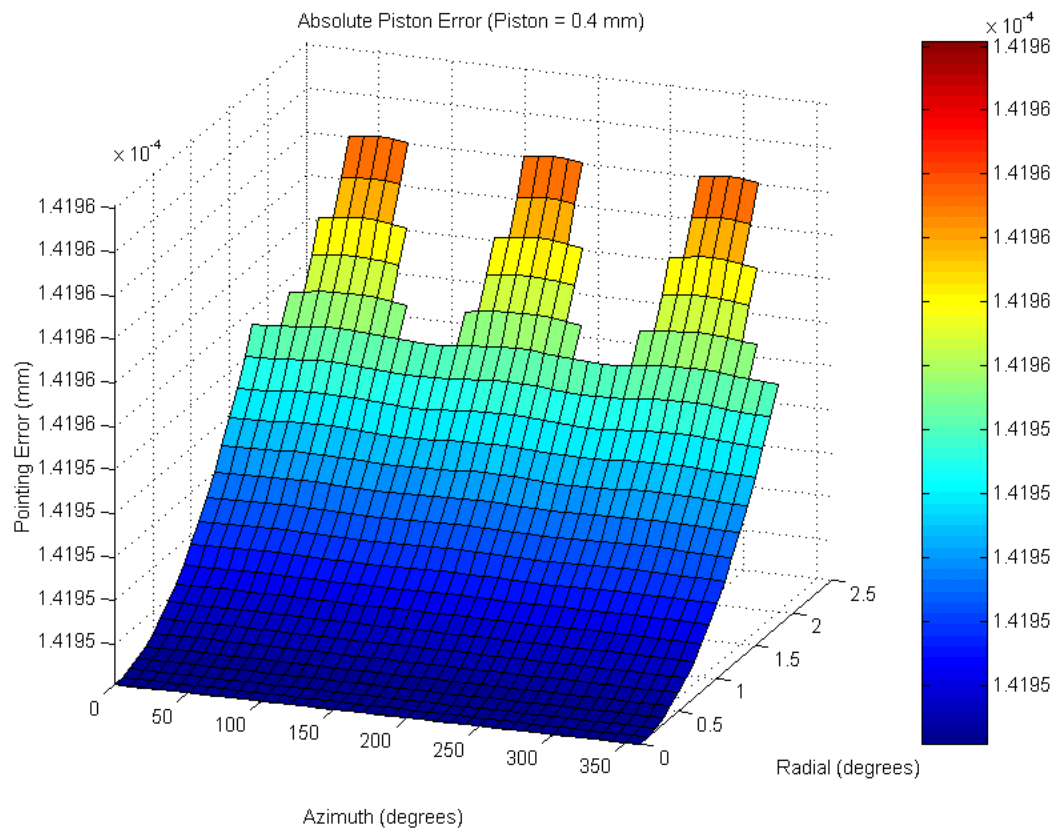
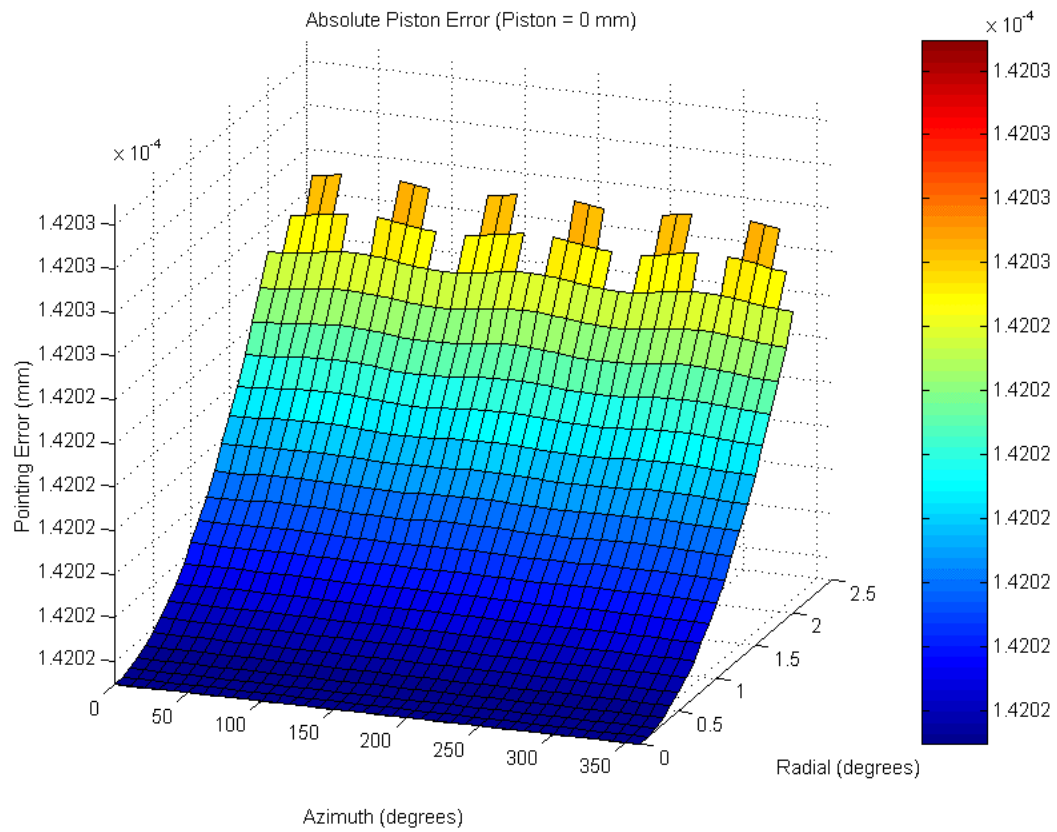
NOTE: These plots represent only a subset of the data presented in the previous table which displays hexapod position error at successive increments of 0.1 mm over the range from -3.4 to $+3.4$ mm in piston. The following plots display data at increments of 0.4 mm over the range from -2.8 to $+2.8$ mm.

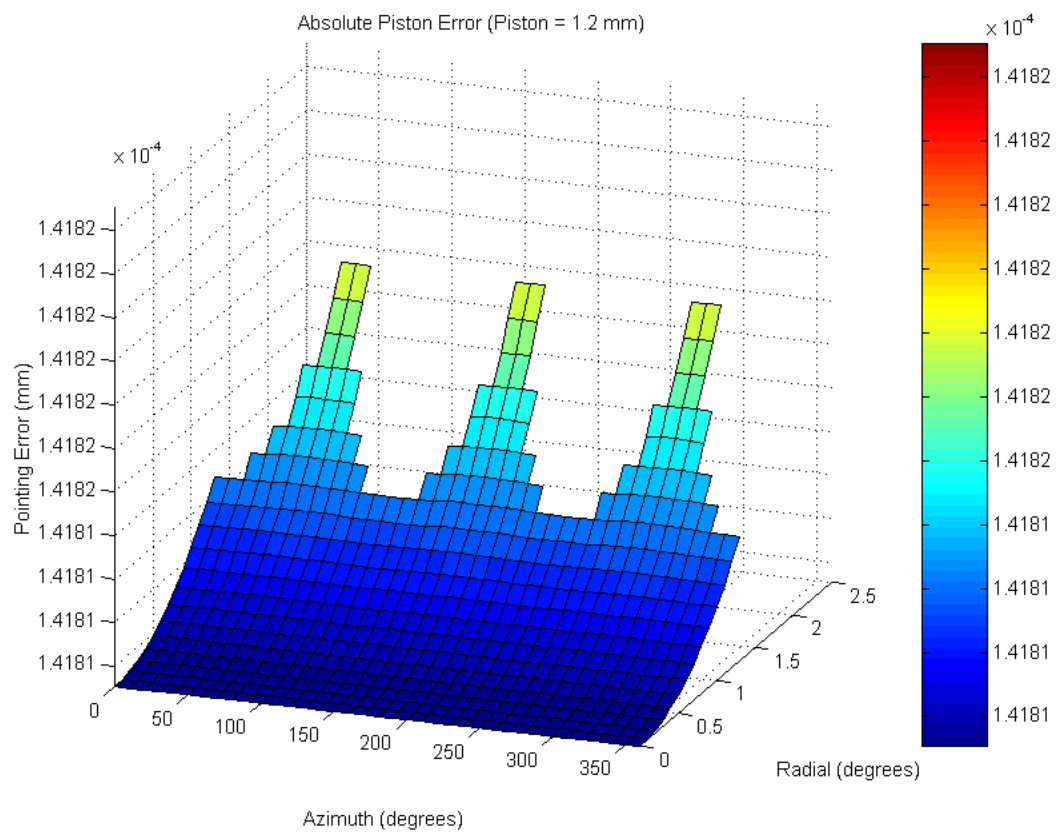
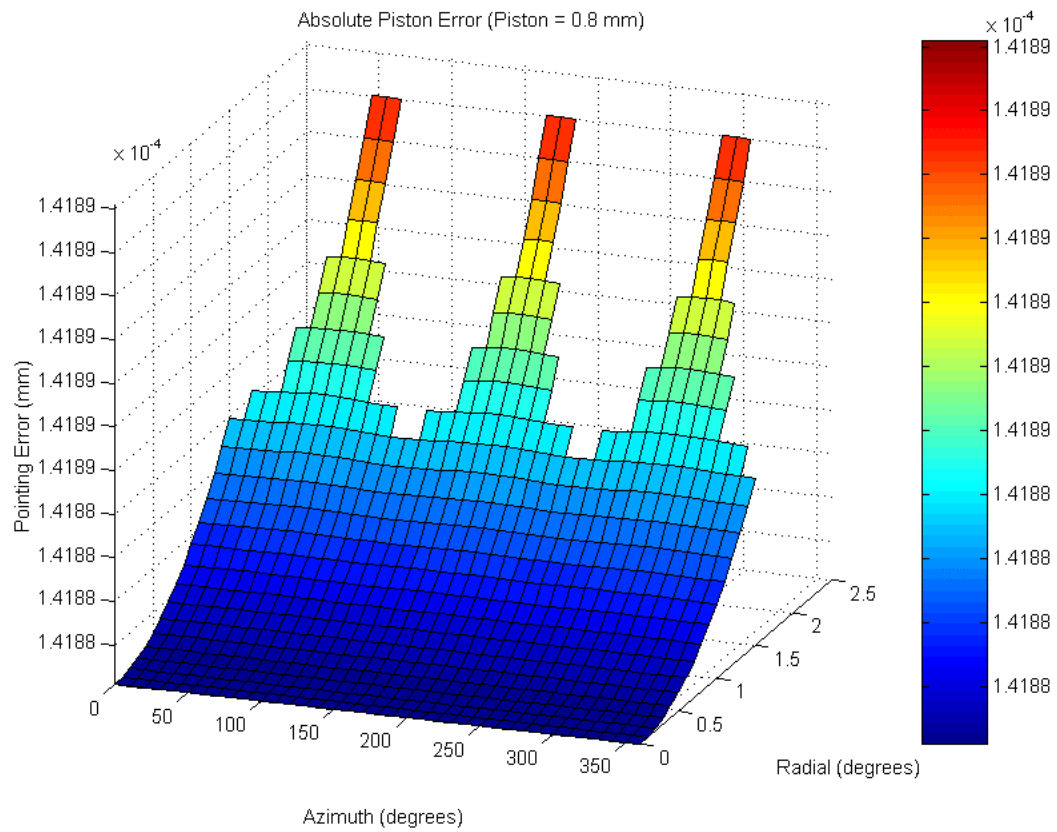












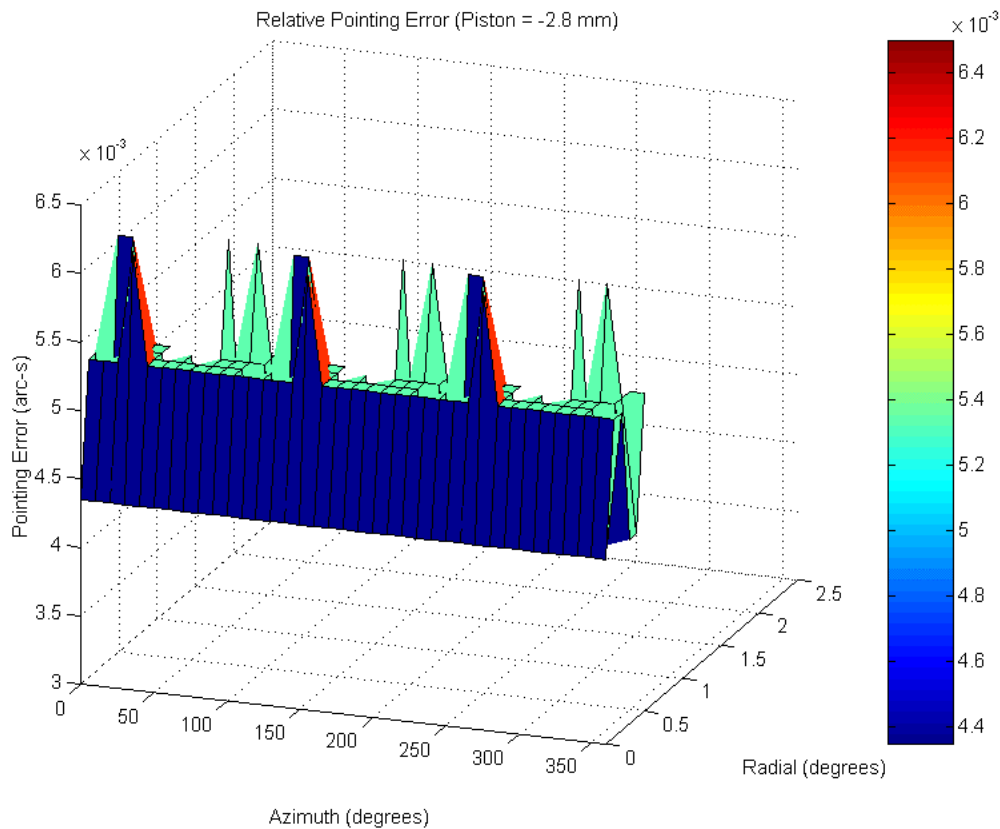
11.0 ANALYTICAL RESULTS – RELATIVE POSITION ACCURACY

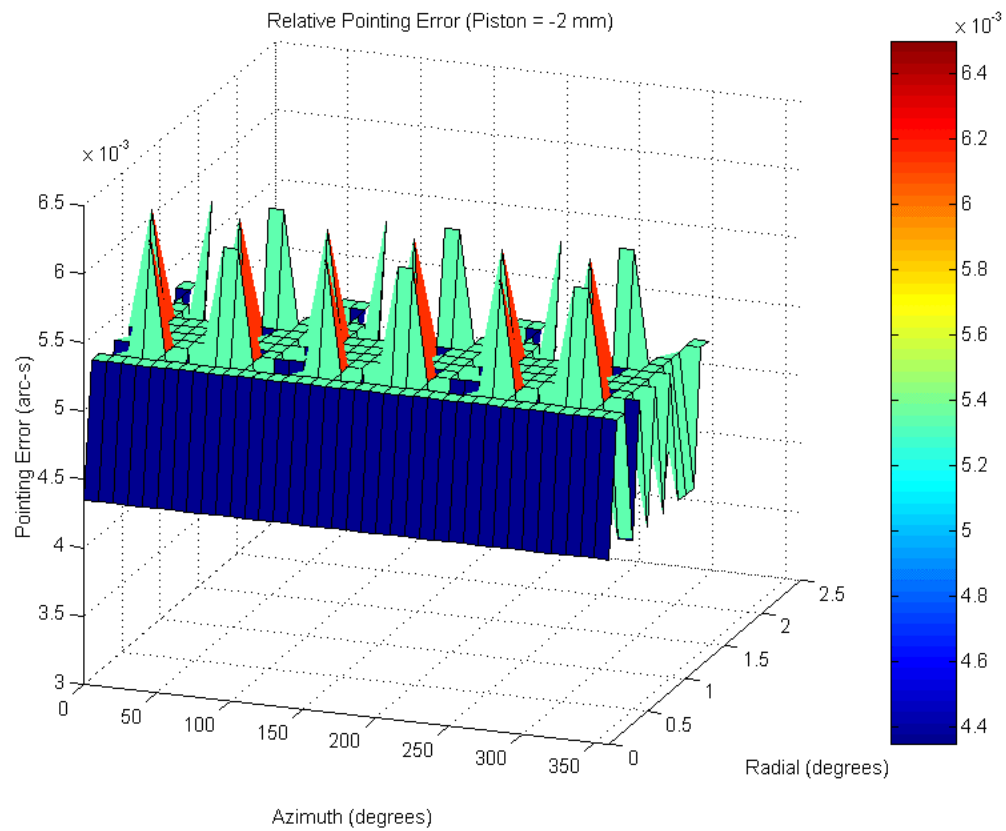
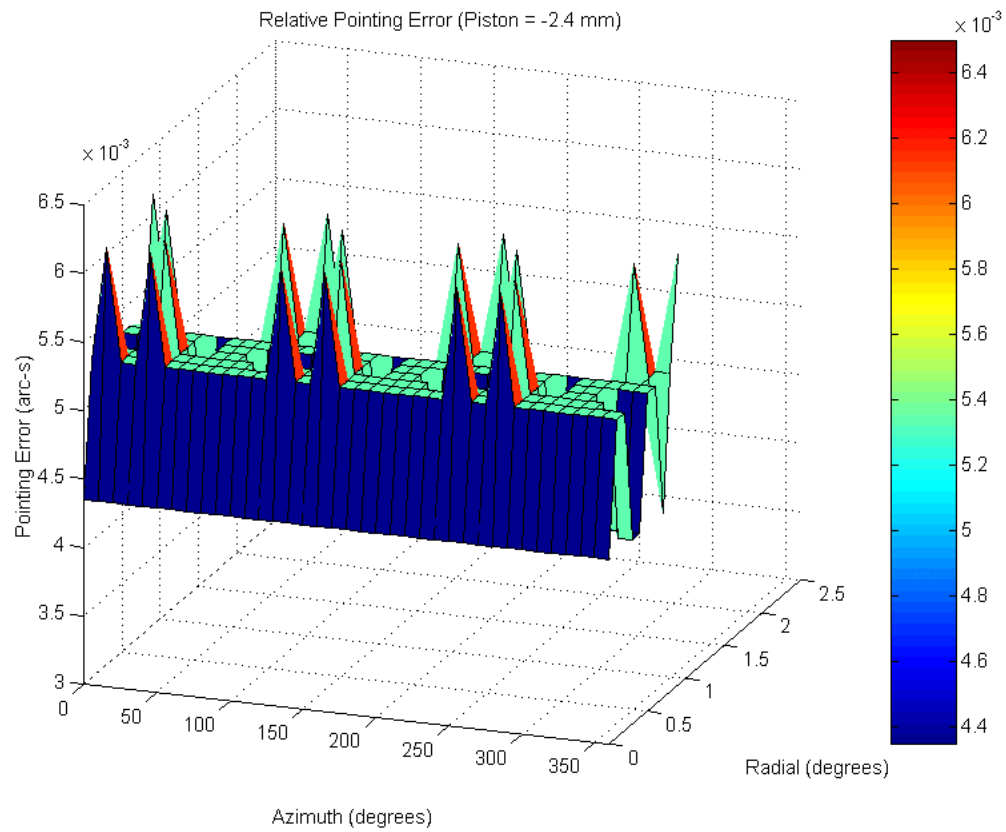
The following table shows the minimum, maximum, and average absolute position error in terms of its constituent components of pointing, decenter, and piston. Each row represents an incremental position of the hexapod in piston. At each increment of piston, the hexapod was then moved through incremental values of radial and azimuth pointing over the full range of motion (at that piston level) to evaluate the hexapod position error. The results are tabulated below.

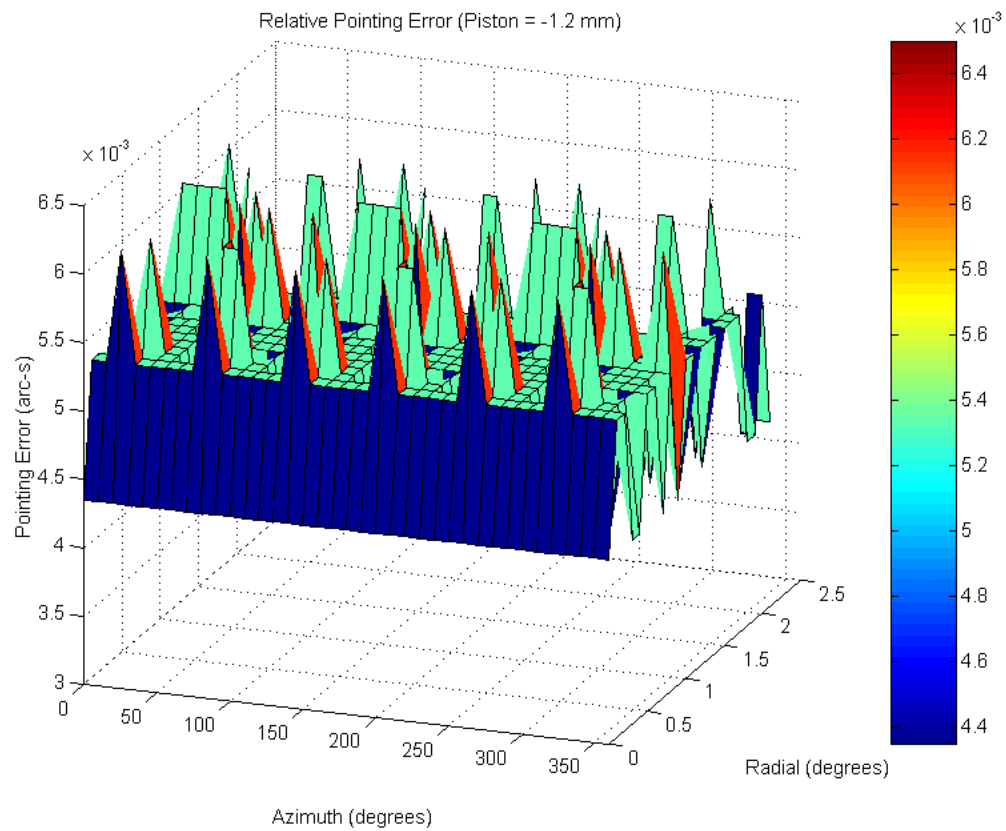
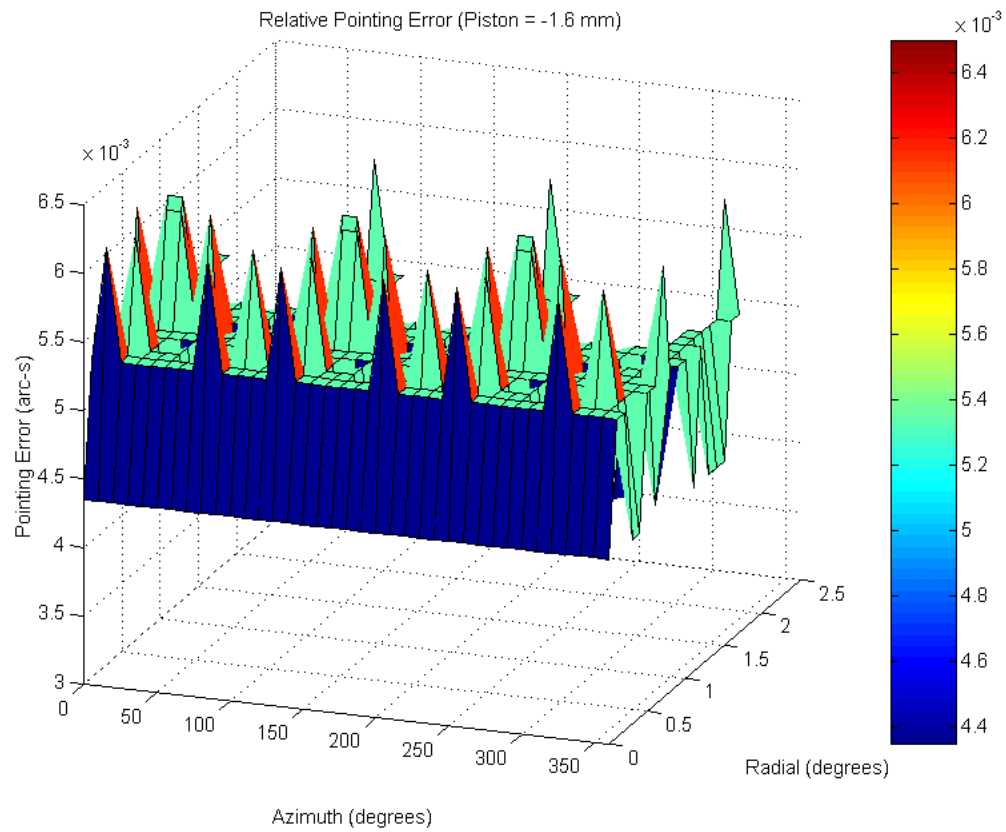
PISTON (mm)	POINTING ERROR			DECENTER ERROR			PISTON ERROR		
	MINIMUM (arc-s)	MAXIMUM (arc-s)	AVERAGE (arc-s)	MINIMUM (mm)	MAXIMUM (mm)	AVERAGE (mm)	MINIMUM (mm)	MAXIMUM (mm)	AVERAGE (mm)
-2.8	4.34671-3	6.14717-3	5.06501-3	8.20689-6	8.21719-6	8.21038-6	6.61275-6	6.61279-6	6.61276-6
-2.4	4.34671-3	6.14717-3	5.13685-3	8.22039-6	8.23793-6	8.22660-6	6.60939-6	6.60951-6	6.60941-6
-2.0	4.34671-3	6.14717-3	5.14424-3	8.23389-6	8.25857-6	8.24258-6	6.60605-6	6.60629-6	6.60609-6
-1.6	4.34671-3	6.14717-3	5.17148-3	8.24739-6	8.27913-6	8.25866-6	6.60273-6	6.60313-6	6.60280-6
-1.2	4.34671-3	6.14717-3	5.17477-3	8.26091-6	8.29685-6	8.27444-6	6.59943-6	6.59995-6	6.59953-6
-0.8	4.34671-3	6.14717-3	5.21552-3	8.27443-6	8.31116-6	8.28977-6	6.59616-6	6.59666-6	6.59627-6
-0.4	4.34671-3	6.14717-3	5.24995-3	8.28796-6	8.32586-6	8.30434-6	6.59290-6	6.59337-6	6.59302-6
+0.0	3.07359-3	6.14717-3	5.10544-3	8.30150-6	8.33801-6	8.31809-6	6.58966-6	6.59009-6	6.58979-6
+0.4	4.34671-3	6.14717-3	5.22642-3	8.31505-6	8.35068-6	8.33107-6	6.58644-6	6.58687-6	6.58656-6
+0.8	4.34671-3	6.14717-3	5.20945-3	8.32860-6	8.36377-6	8.34346-6	6.58324-6	6.58375-6	6.58334-6
+1.2	3.07359-3	6.14717-3	5.14718-3	8.34216-6	8.37567-6	8.35506-6	6.58005-6	6.58056-6	6.58014-6
+1.6	4.34671-3	6.14717-3	5.18168-3	8.35573-6	8.38392-6	8.36629-6	6.57689-6	6.57724-6	6.57695-6
+2.0	4.34671-3	6.14717-3	5.17974-3	8.36930-6	8.39083-6	8.37755-6	6.57375-6	6.57395-6	6.57378-6
+2.4	4.34671-3	6.14717-3	5.09150-3	8.38288-6	8.39905-6	8.38874-6	6.57062-6	6.57074-6	6.57064-6
+2.8	4.34671-3	5.32361-3	5.02629-3	8.39647-6	8.40592-6	8.39972-6	6.56752-6	6.56756-6	6.56752-6
Total	3.07359-3	6.14717-3	5.17332-3	8.20689-6	8.40592-6	8.31360-6	6.56752-6	6.61279-6	6.59001-6

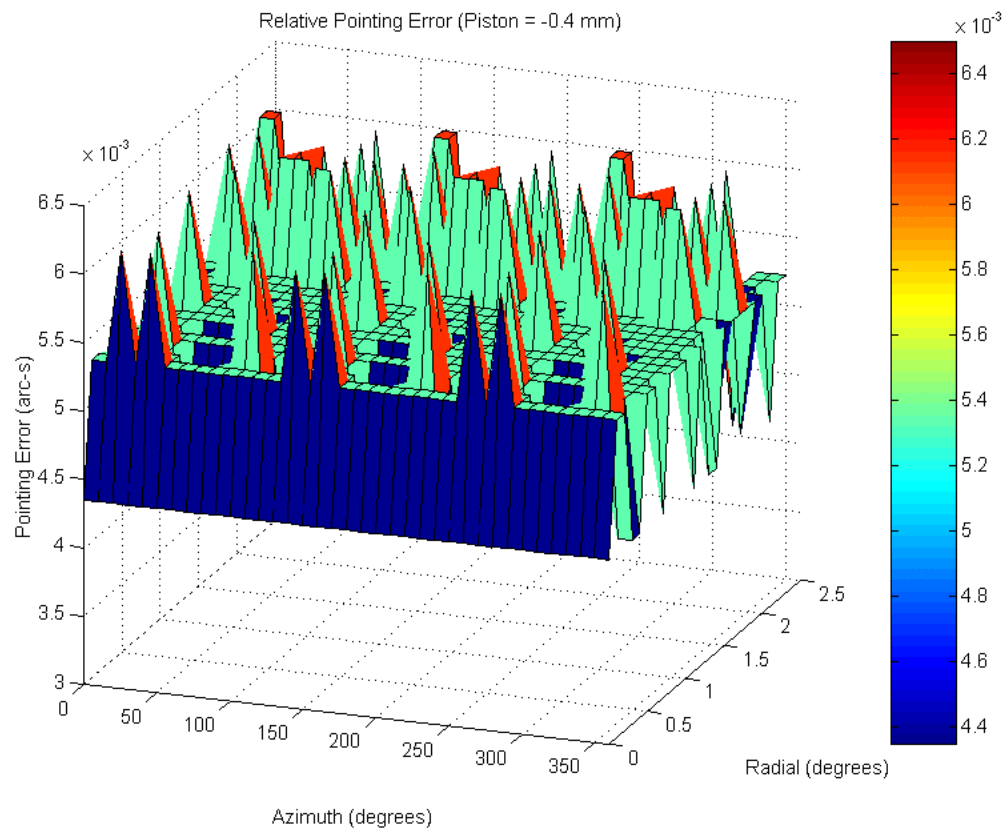
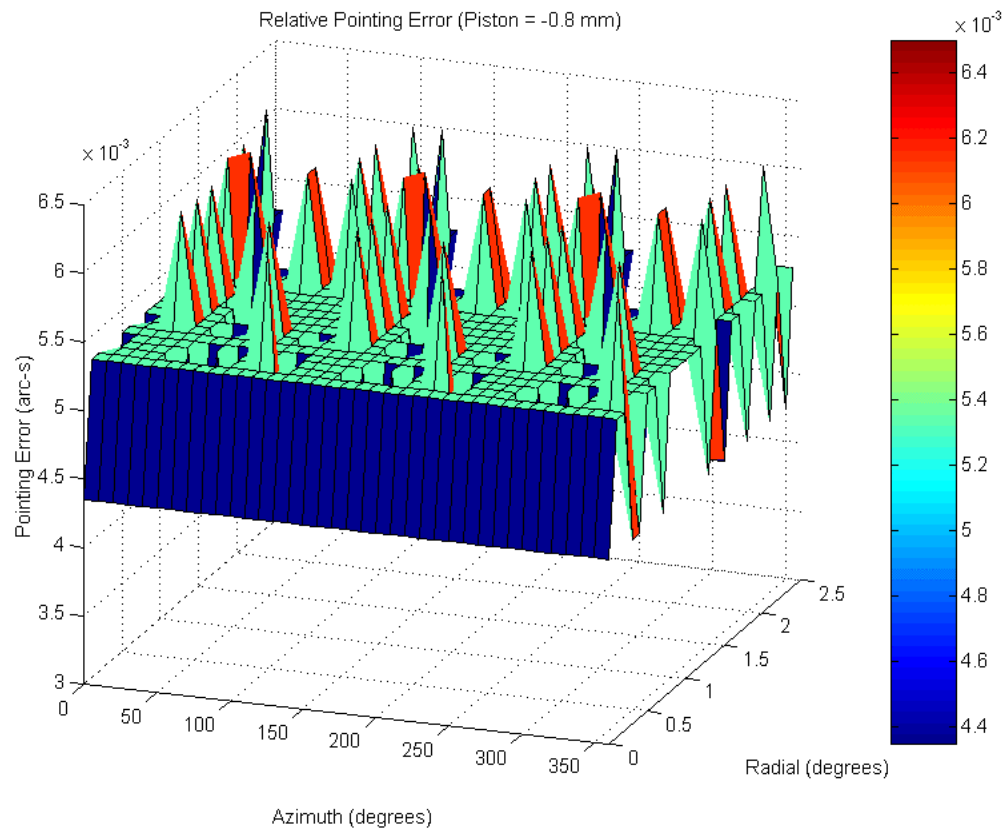
RELATIVE POINTING ERROR MAPS OF THE NOMINAL HEXAPOD DESIGN

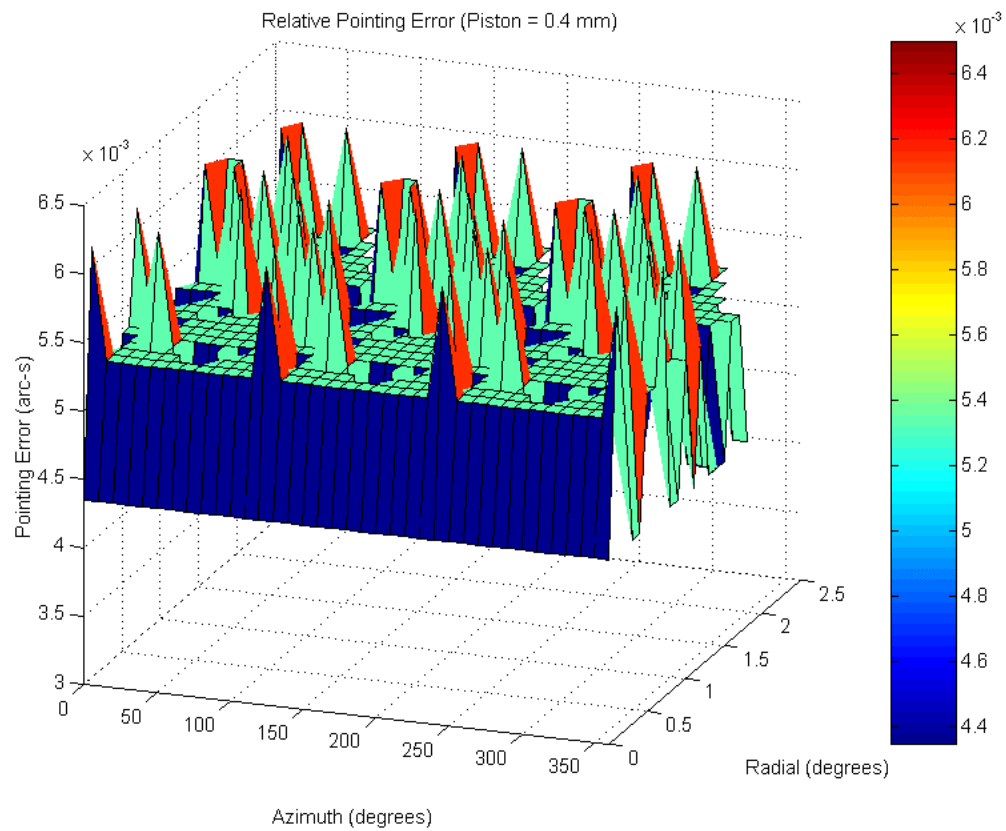
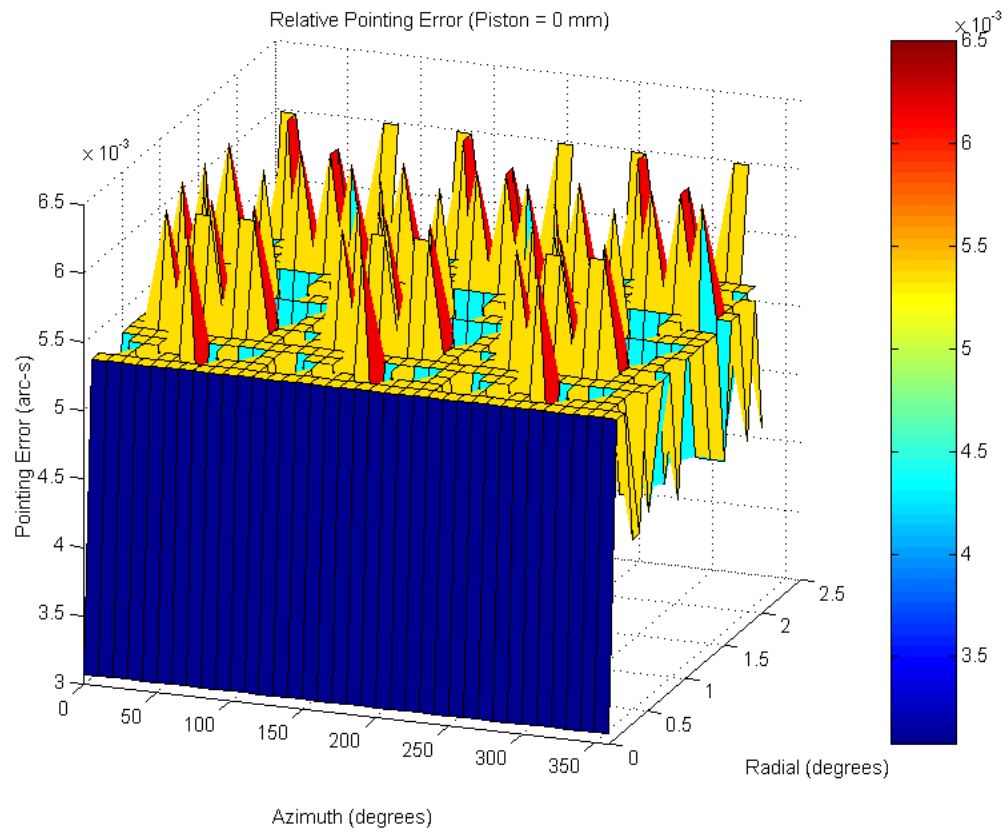
The following series of plots show the relative pointing accuracy over the entire operational range of the nominal hexapod design. Each plot corresponds to a single value of piston at which the hexapod was then moved in pure rotations of radial and azimuth pointing to complete the plot. All plots in the series are presented with the same total range for the pointing error (along the vertical axis) so that all data is uniformly scaled from one plot to the next. The ensuing plots show the relative pointing accuracy to be better than $5.1\text{E-}3$ arc-s over the entire operational range. Since $5.1\text{E-}3$ arc-s is the smallest relative pointing angle that can be resolved by the analysis software, this is the value that is plotted when the actual pointing accuracy exceeds $5.1\text{E-}3$ arc-s. In the ensuing plots, the positive and negative spikes are caused by numerical errors.

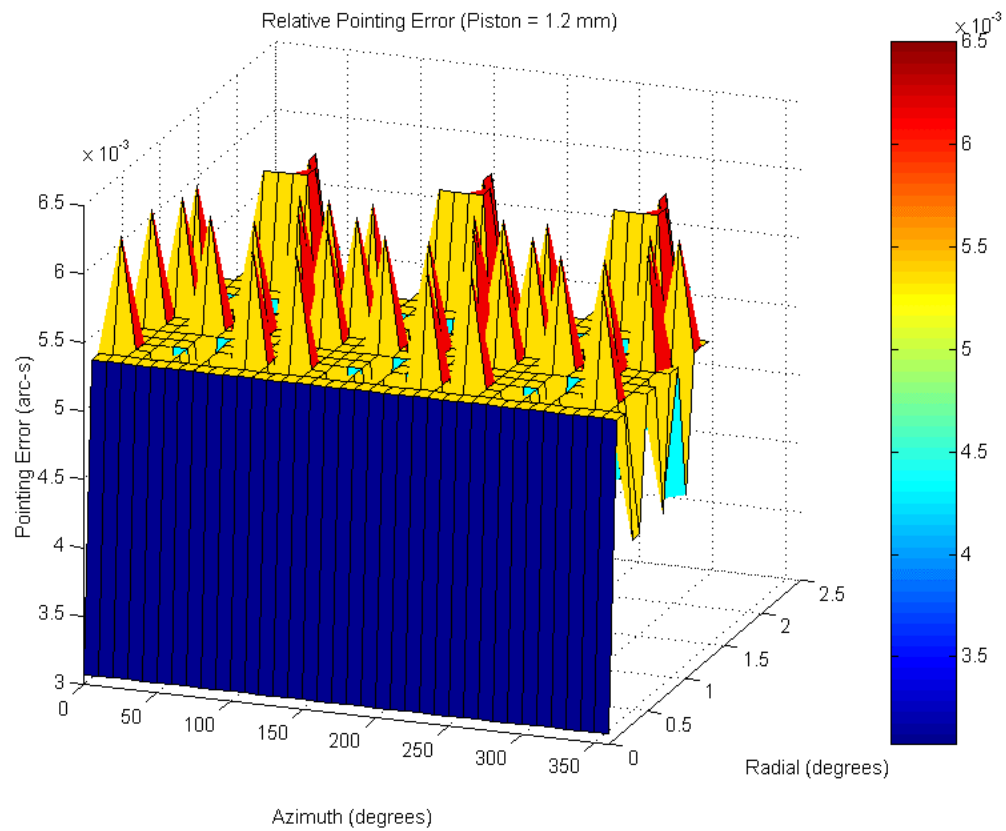
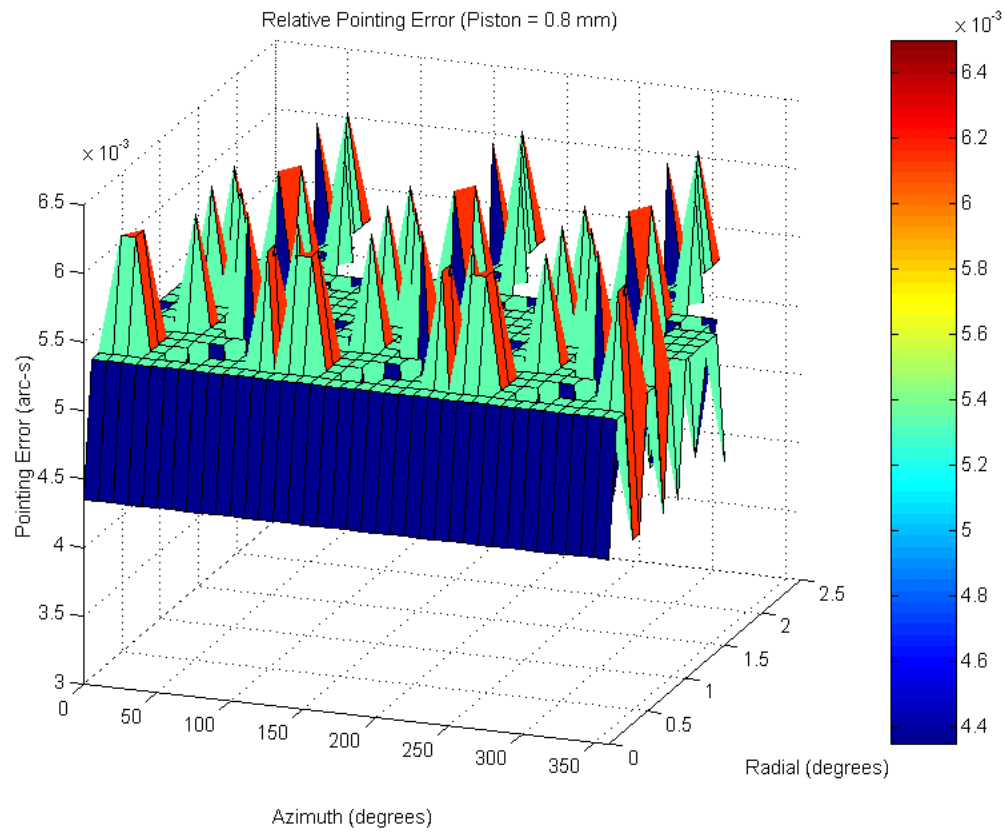


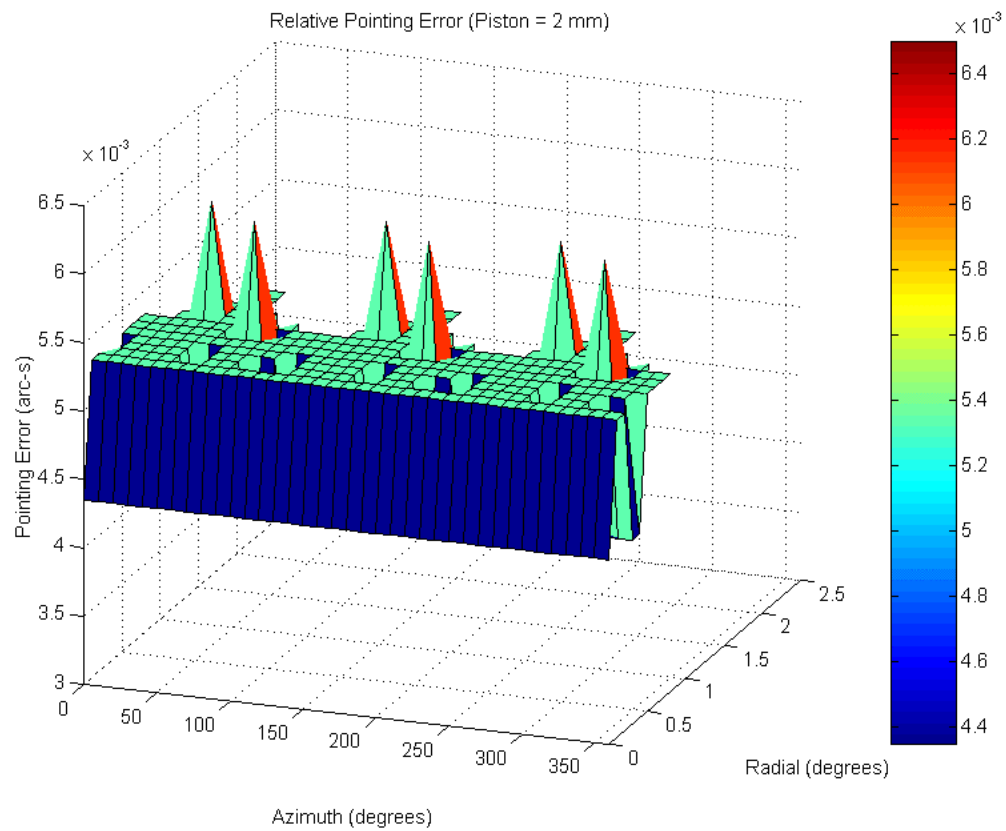
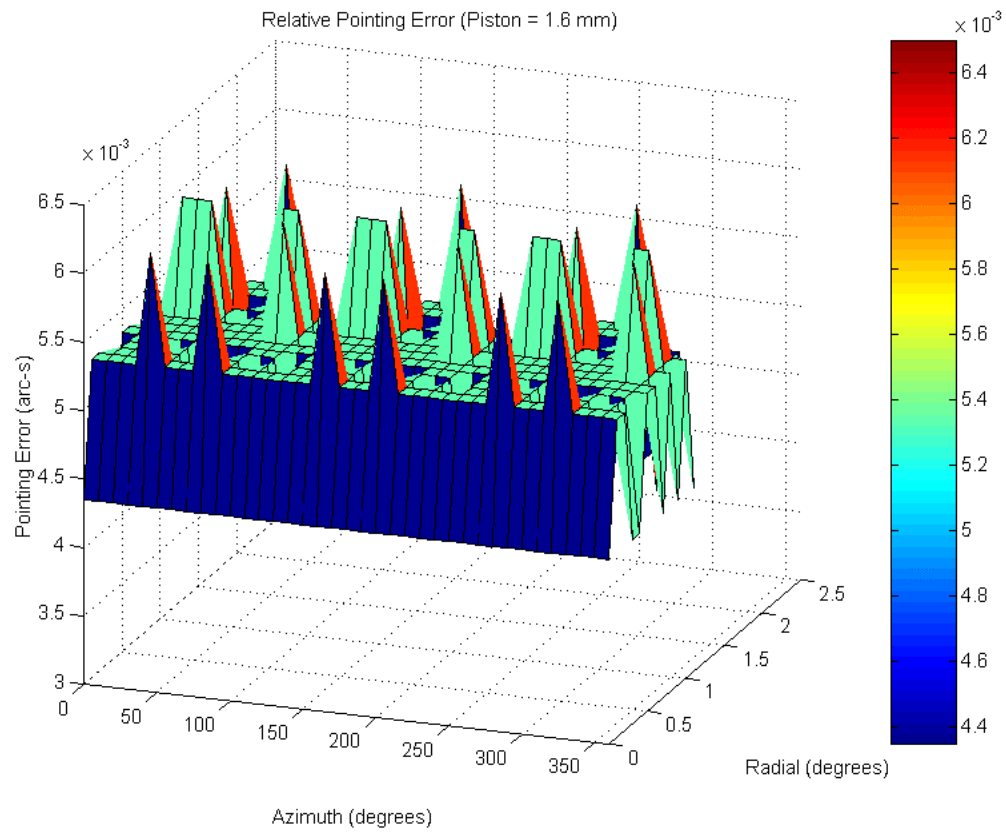


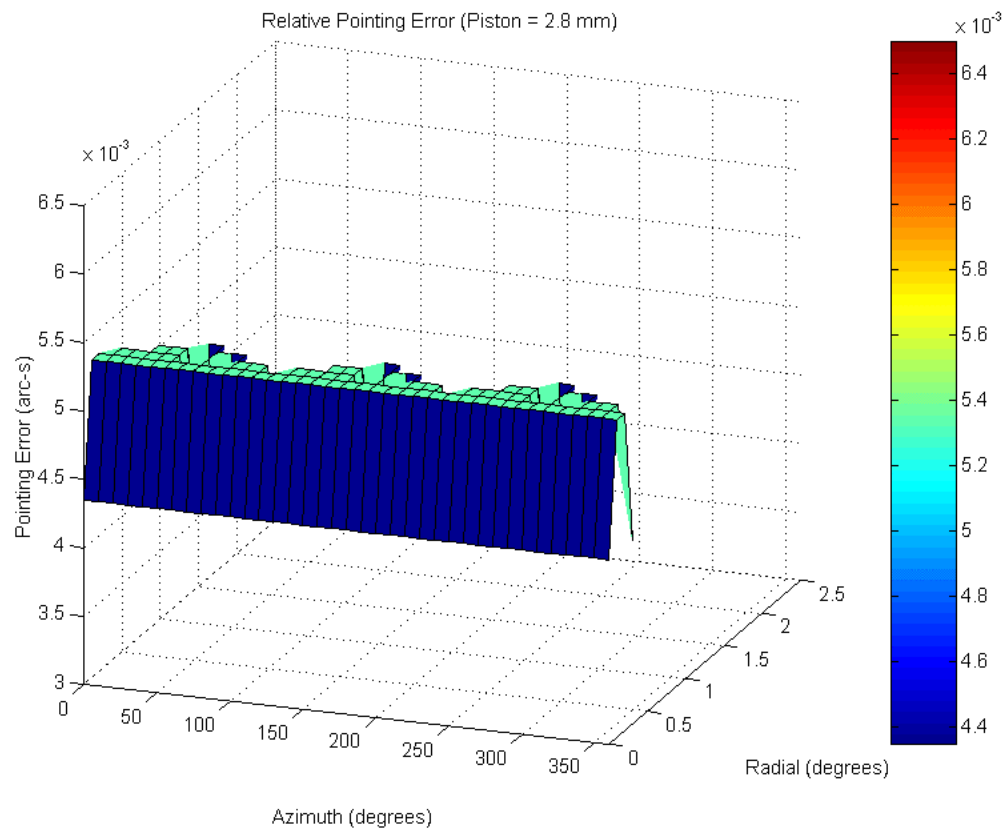
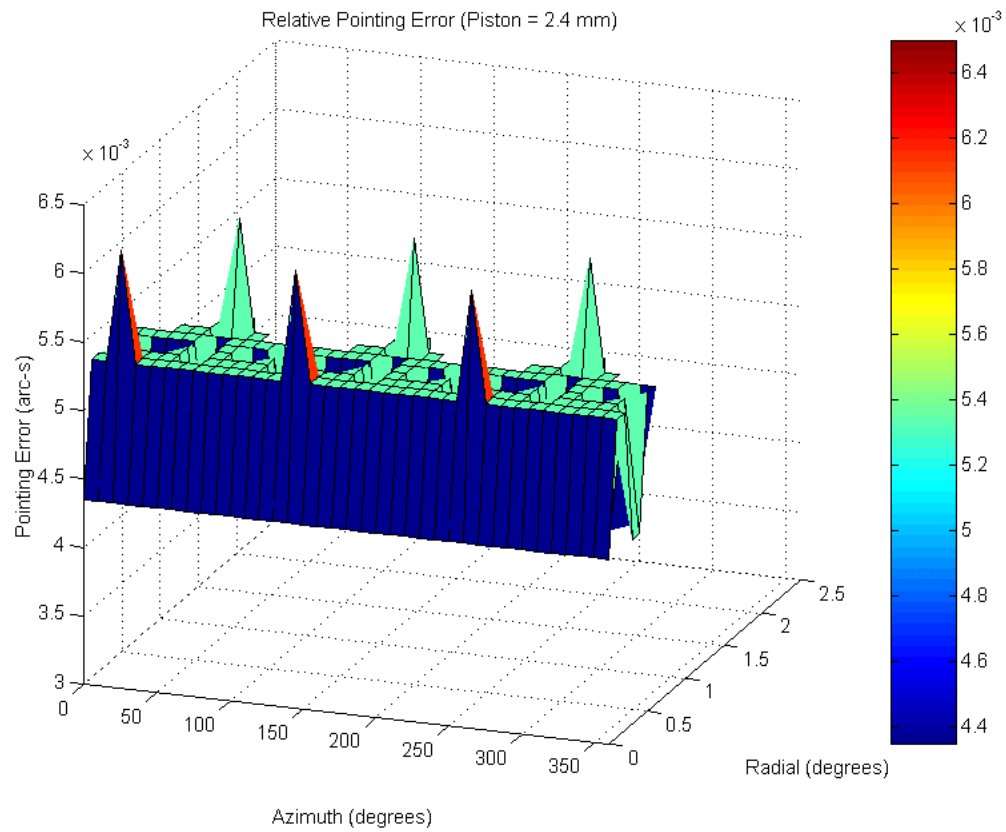






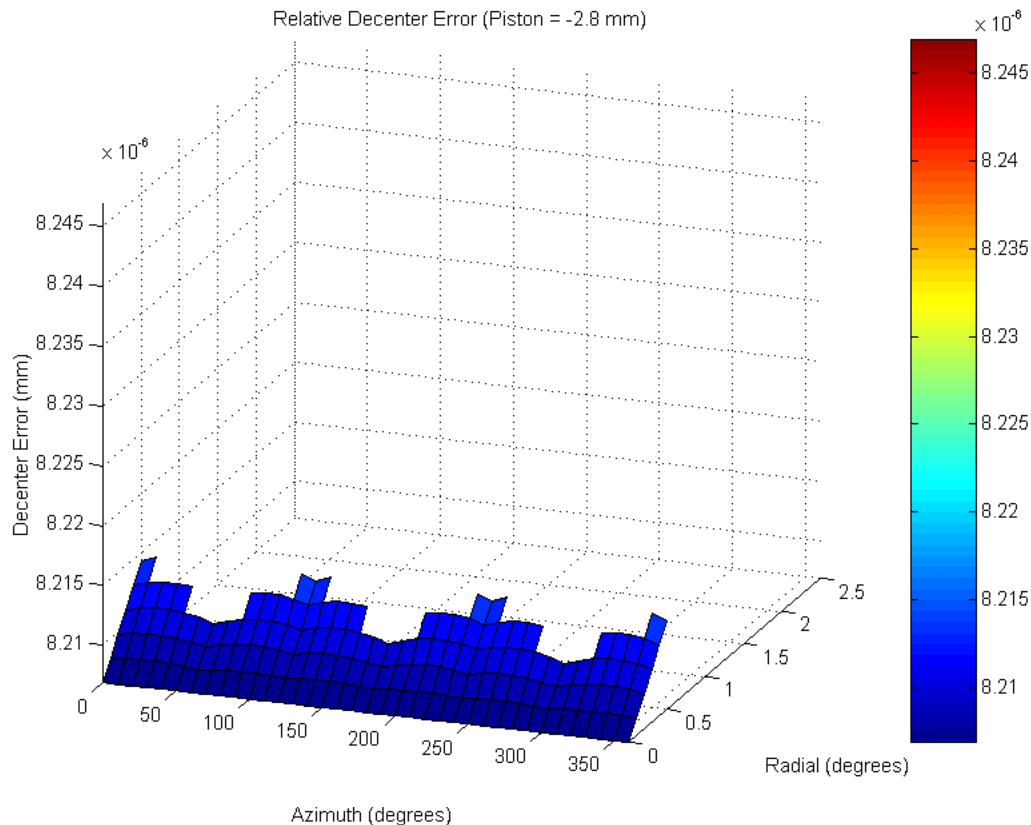


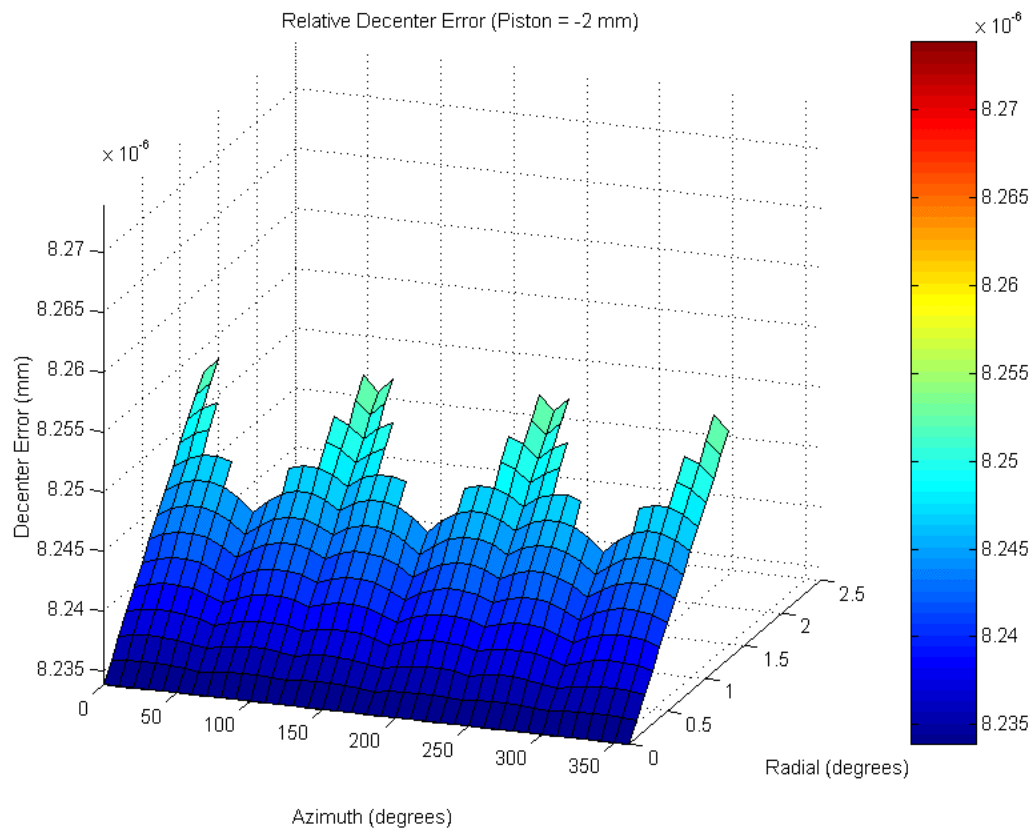
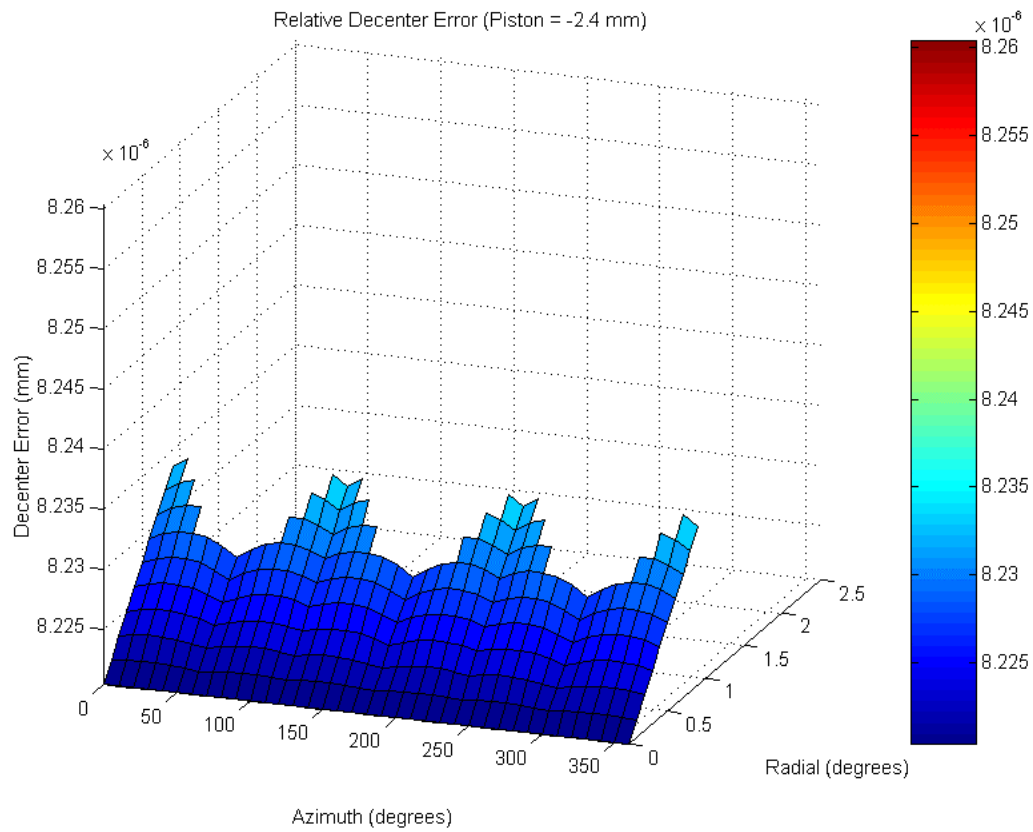


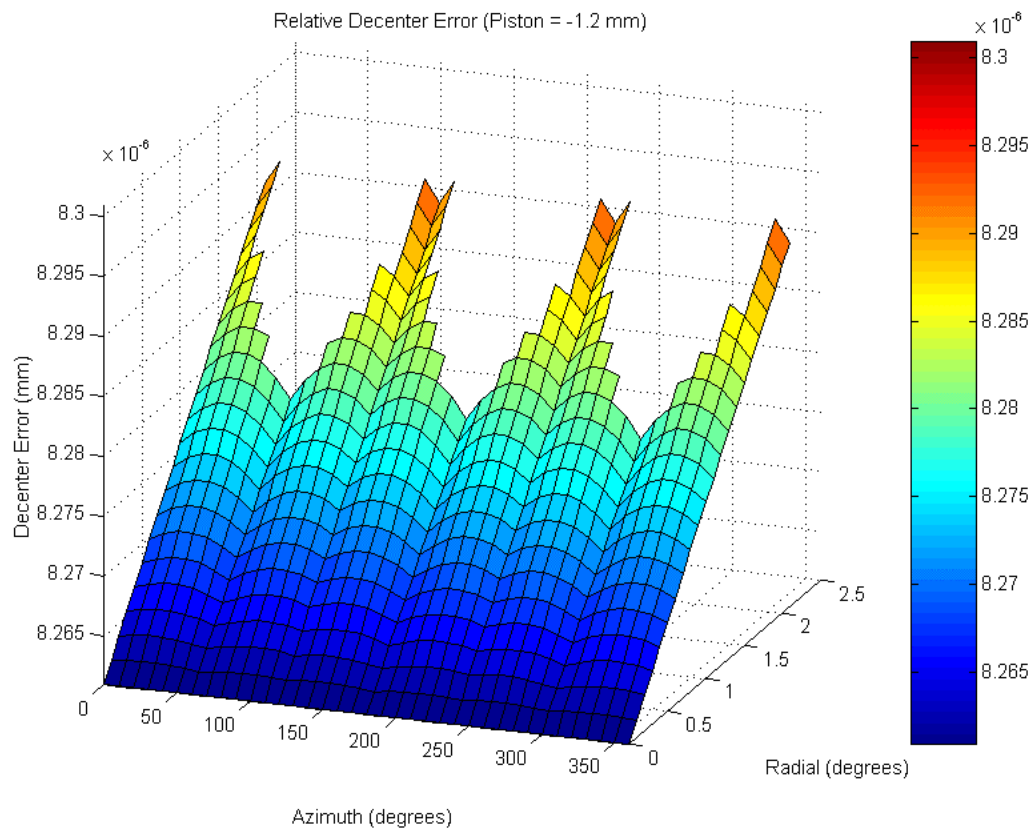
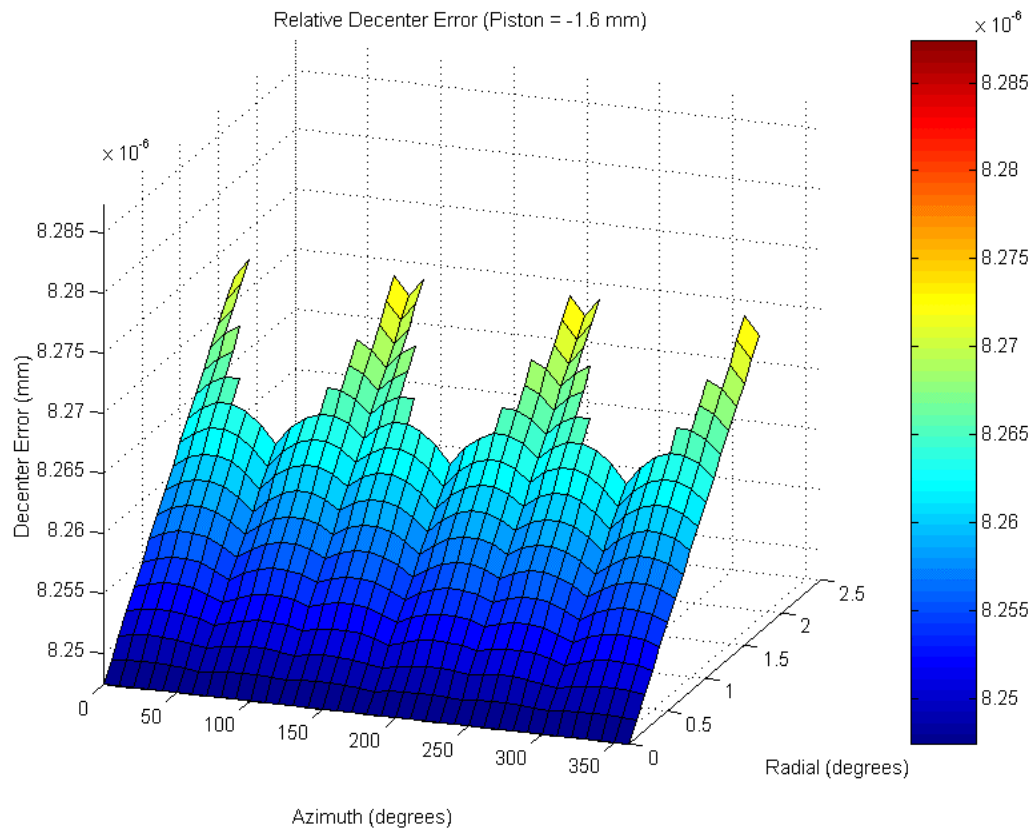


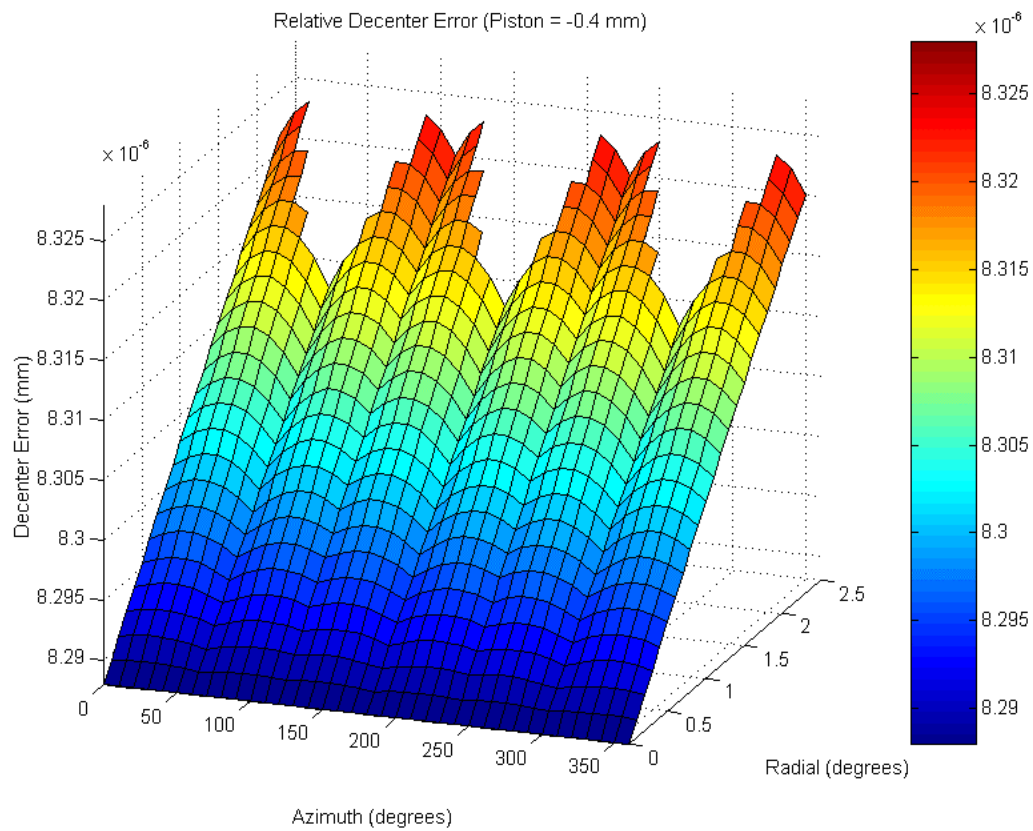
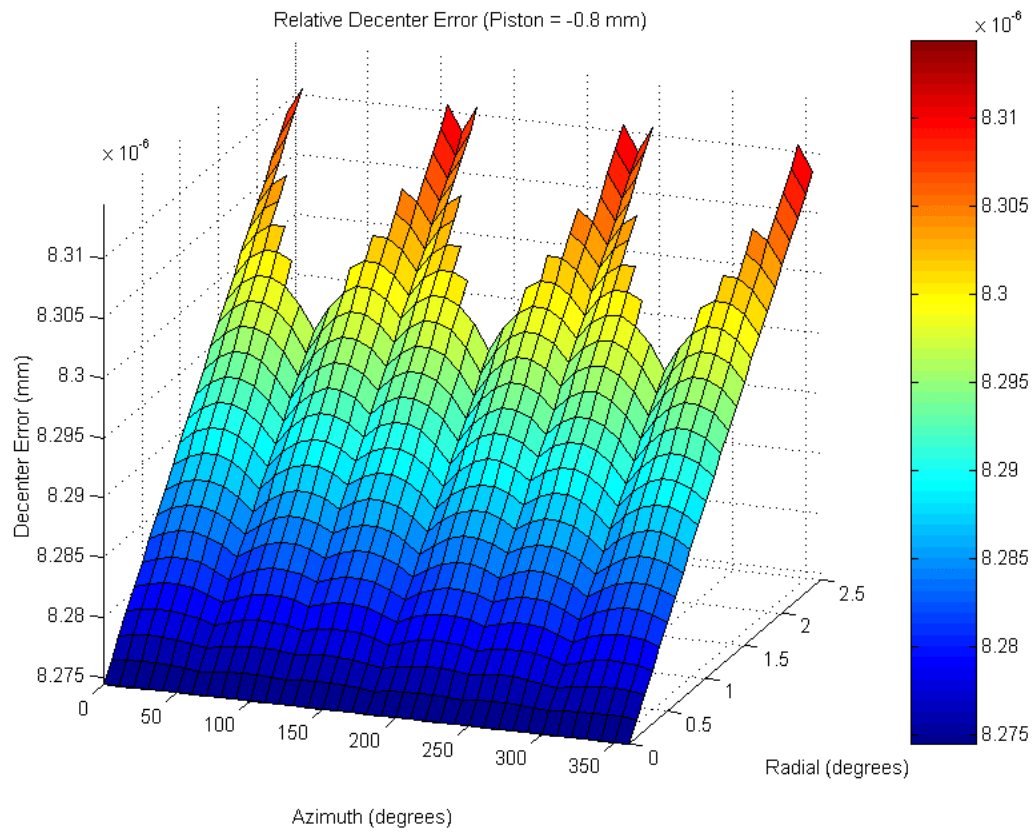
RELATIVE DECENTER ERROR MAPS OF THE NOMINAL HEXAPOD DESIGN

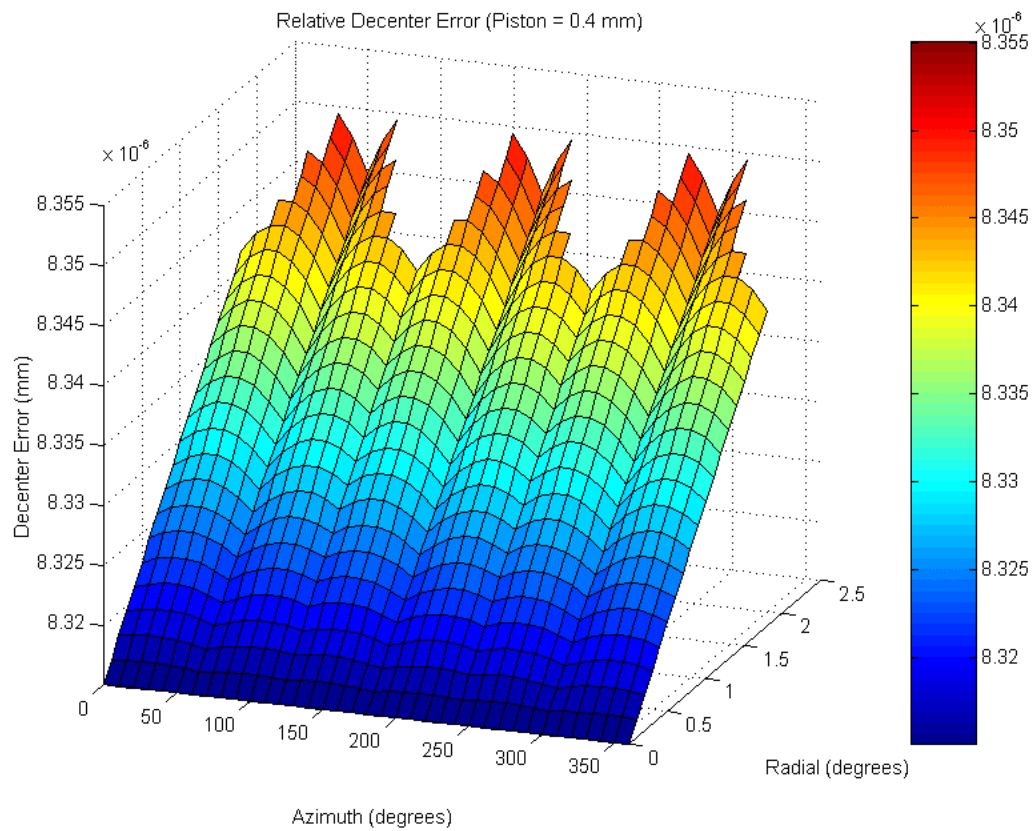
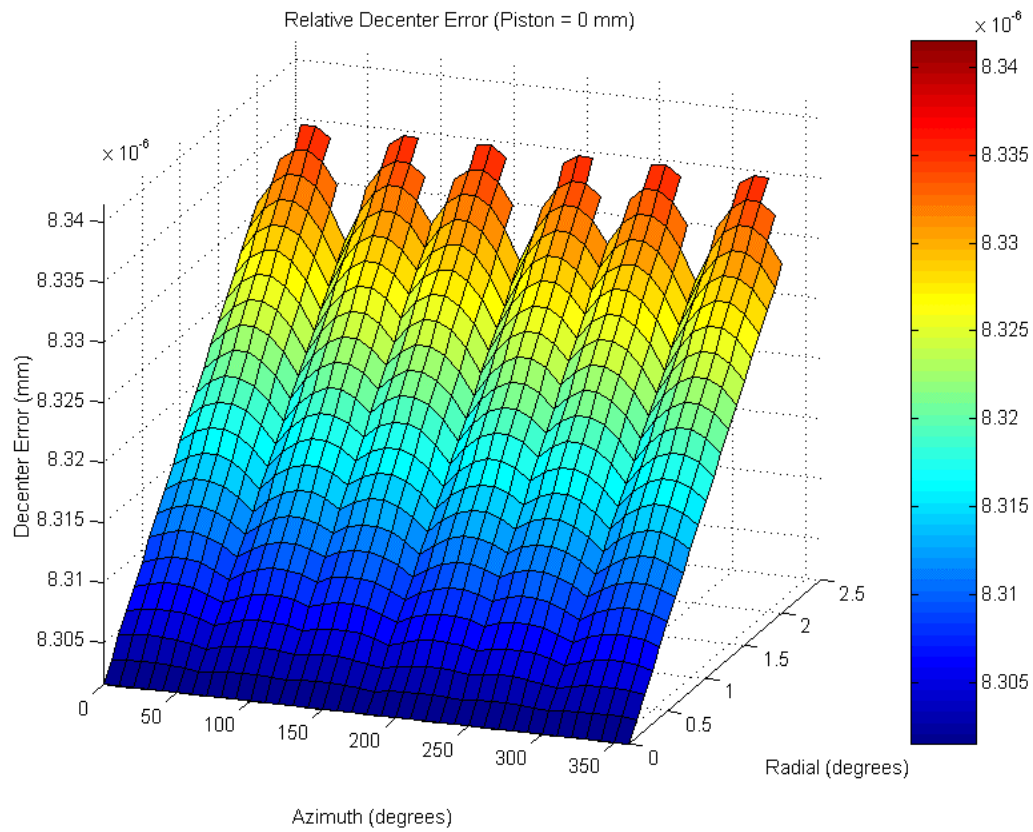
The following series of plots show the relative decenter error over the entire operational range of the nominal hexapod design. Each plot corresponds to a single value of piston at which the hexapod was then moved in pure rotations of radial and azimuth pointing to complete the plot. All plots in the series are presented with the same total range for the decenter error (along the vertical axis) so that all data is uniformly scaled from one plot to the next. However, it should be noted that the minimum and maximum range extents of the vertical axis tend to shift upward in each successive plot. This shift reflects the fact that the decenter error varies directly as a function of the hexapod's overall height. It should be noted that the relative decenter error has roughly an equal dependence on the radial pointing angle of the hexapod as well. In simple terms, the decenter error tends to increase as any one or more of the hexapod's actuators move towards maximum extension. It should also be noted that there is a moderate dependence of the decenter error on the azimuth-pointing angle of the hexapod, and this dependence is responsible for the three lobed, repeating pattern that is readily seen in all the plots.

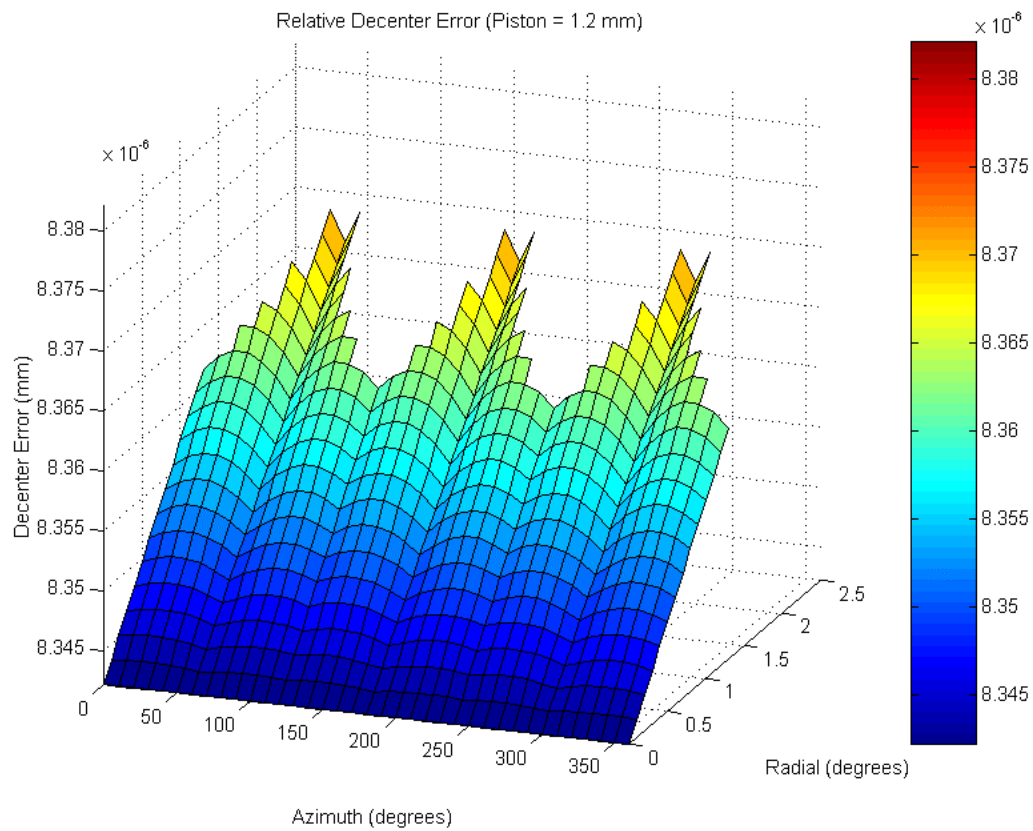
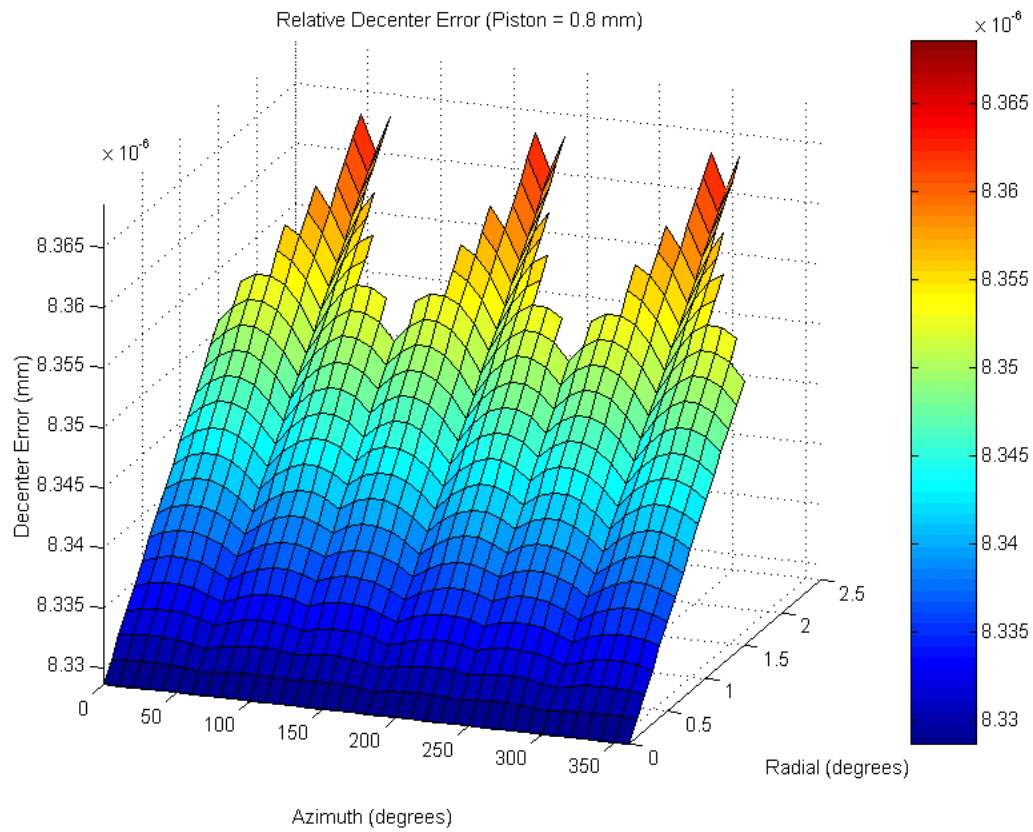


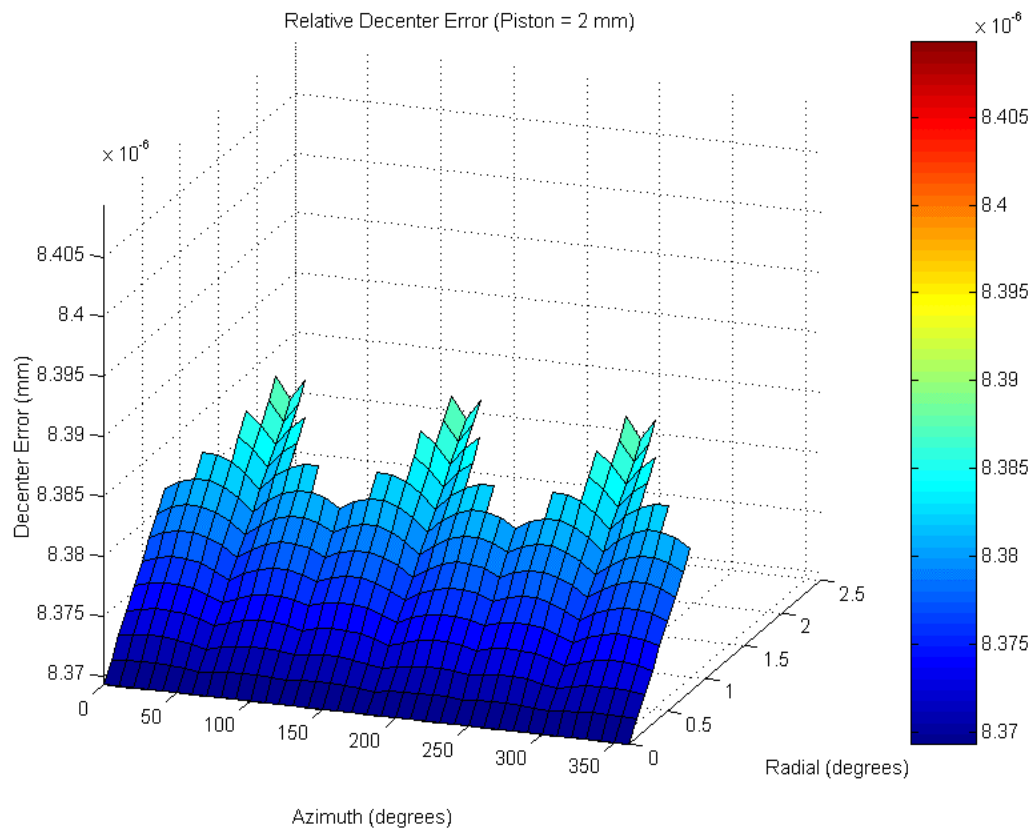
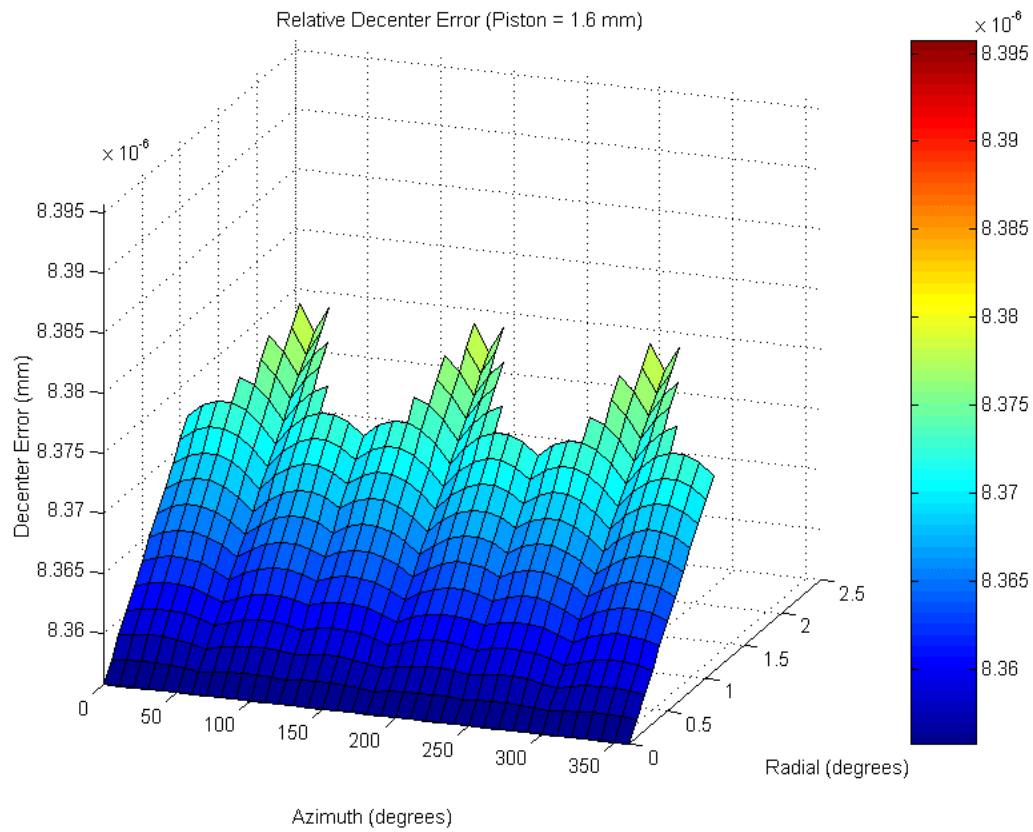


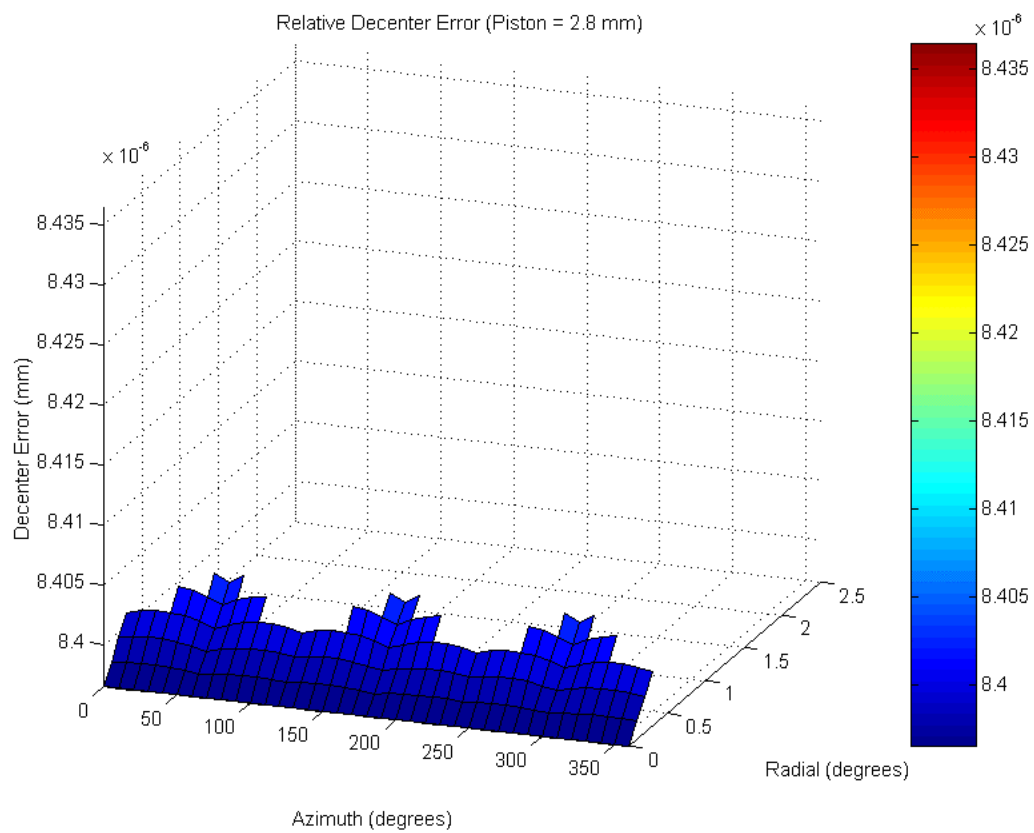
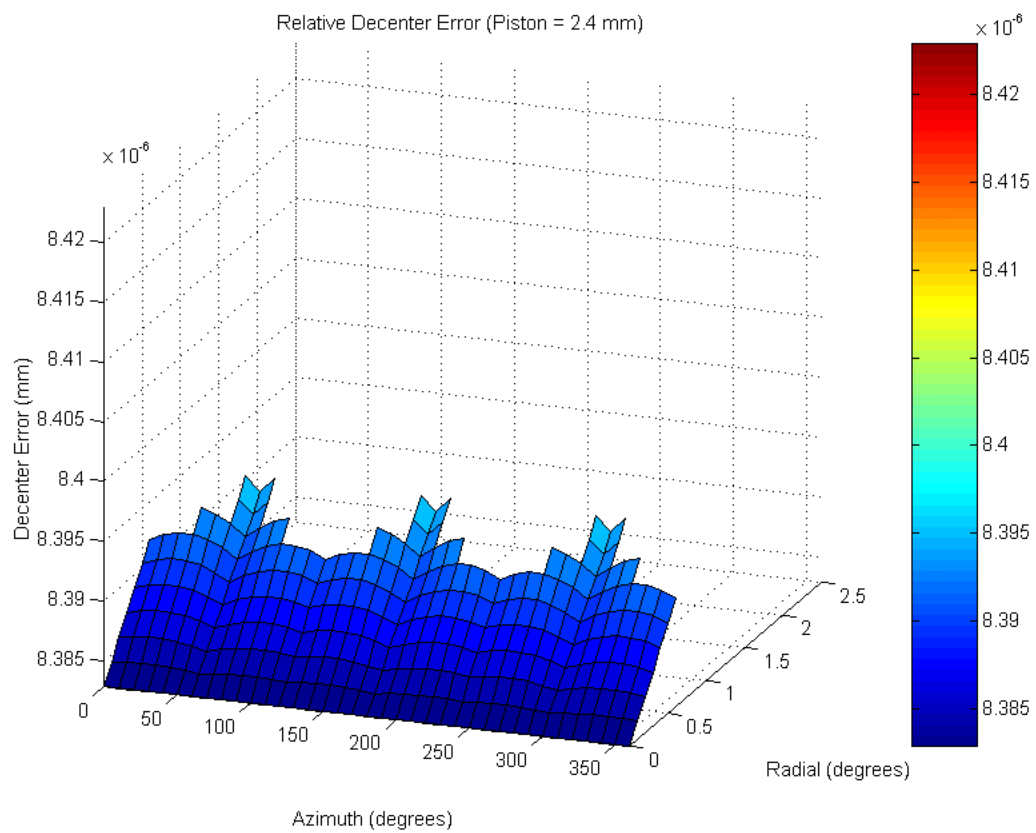












RELATIVE PISTON ERROR MAPS OF THE NOMINAL HEXAPOD DESIGN

Of all the three component error sources that were evaluated, the piston error is the easiest to understand. This is because it's almost entirely a function of the hexapod's overall height. When reviewing single plots of the piston error, this is not readily apparent because each plot shows a definite increase in piston error as a function of radial pointing angle. However, it should be noted that the range of the piston error on a single plot is much smaller than the total range of piston error over all the plots combined. Since each successive plot represents an increase in the overall height of the hexapod, the predominant factor in determining the absolute piston error is the height. For the sake of completeness, it should also be noted that there is an extremely small dependence of the piston error on the azimuth-pointing angle of the hexapod as well. This shows up as a slight variation (or waviness) in the individual plots and is most noticeable in the plot corresponding to 0 mm of piston.

The following series of plots show the piston accuracy over the entire operational range of the nominal hexapod design. Each plot corresponds to a single value of piston at which the hexapod was then moved in pure rotations of radial and azimuth pointing to complete the plot. All plots in the series are presented with the same total range for the pointing error (along the vertical axis) so that all data is uniformly scaled from one plot to the next. However, it should be noted that the minimum and maximum range extents of the vertical axis tend to shift upward in each successive plot. This shift reflects the fact that the pointing error varies directly as a function of the hexapod's overall height (as previously mentioned).

